1p 21 – clasa a IX-a Z 15 – 09

A. (15)

$$\begin{array}{l|l} & a, b, c, d \div \\ \text{I} & a+1,\ b+1,\ c+3,\ d+9 \ \div\div \\ \hline \text{C} & \text{aflați } a, b, c, d. \end{array}$$

$a, b, c, d \div$ cu rația $r \Rightarrow b = a+r,\ c = a+2r,\ d = a+3r \Rightarrow$

$$\Rightarrow a+1,\ a+r+1,\ a+2r+3,\ a+3r+9 \ \div\div \Rightarrow \begin{cases} (a+r+1)^2 = (a+1)(a+2r+3) \\ (a+2r+3)^2 = (a+r+1)(a+3r+9) \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} \cancel{a^2} + r^2 + 1 + \cancel{2ar} + 2a + \cancel{2r} = \cancel{a^2} + \cancel{2ar} + 3a + a + \cancel{2r} + 3 \\ \cancel{a^2} + 4r^2 + 9 + \cancel{4ar} + 6a + \cancel{12r} = \cancel{a^2} + \cancel{4ar} + 10a + 3r^2 + \cancel{12r} + 9 \end{cases}$$

$$\Rightarrow \begin{cases} r^2 = 2a+2 \\ r^2 = 4a \end{cases} \Rightarrow \begin{cases} 2a+2 = 4a \Rightarrow \boxed{a = +1} \\ r^2 = 4a \end{cases} \Rightarrow r^2 = 4 \Rightarrow r = \pm 2$$

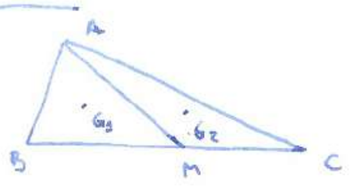
pt $r = 2$: $(a, b, c, d) = (1, 3, 5, 7)$

pt $r = -2$: $(a, b, c, d) = (1, -1, -3, -5)$ ■

Temă:

$$\begin{array}{l|l} & a, b, c \div \\ & a+b+c = 21 \\ \text{I.} & a+3,\ b+5,\ c+13 \ \div\div \\ \hline \text{C.} & (a, b, c) = ? \end{array}$$

G. (15) Fie $M \in (BC)$ iar G_1, G_2 cdg ale $\triangle ABM$ resp $\triangle AMC$ (A, B, C necoliniare). Să se arate că $\overline{BC}$ și $\overline{G_1G_2}$ sunt coliniari.



$$\overline{G_1G_2} = \overline{AG_2} - \overline{AG_1} = \frac{1}{3}\left(\overline{AA} + \overline{AC} + \cancel{\overline{AM}} - \overline{AA} - \overline{AB} - \cancel{\overline{AM}}\right) =$$

$$= \frac{1}{3}\left(\overline{AC} - \overline{AB}\right) = \frac{1}{3}\overline{BC} \Rightarrow \overline{G_1G_2} \parallel \overline{BC}$$ ■

Temă: ABCD patrulater convex cu G_1 cdg al $\triangle ABD$ și G_2 cdg al $\triangle ABC$.

Atunci $\overline{G_1G_2} \parallel \overline{CD}$.

T. (15) Rezolvați ecuația: $\arcsin(3x) + \arcsin(4x) = \arcsin(5x)$ (*)

CE: $\left.\begin{array}{l} 3x \in [-1,1] \\ 4x \in [-1,1] \\ 5x \in [-1,1] \end{array}\right\} \Rightarrow x \in \left[-\frac{1}{5}, \frac{1}{5}\right]$

$(*) \Rightarrow \sin(\arcsin(3x) + \arcsin(4x)) = \sin(\arcsin(5x)) \Leftrightarrow$

$\Leftrightarrow \sin(\arcsin(3x)) \cdot \cos(\arcsin(4x)) + \sin(\arcsin(4x)) \cdot \cos(\arcsin(3x)) = 5x \Leftrightarrow$

$\Leftrightarrow 3x \cdot \sqrt{1-(4x)^2} + 4x \cdot \sqrt{1-(3x)^2} = 5x \Leftrightarrow$

$\Leftrightarrow x\left[3\sqrt{1-16x^2} + 4\sqrt{1-9x^2}\right] = 5x \Leftrightarrow x = 0 \in \left[-\frac{1}{5}, \frac{1}{5}\right]$ soluție

sau $3\sqrt{1-16x^2} + 4\sqrt{1-9x^2} = 5 \Leftrightarrow$

$\Leftrightarrow 3\sqrt{1-16x^2} = 5 - 4\sqrt{1-9x^2} \overset{(\,)^2}{\Rightarrow} 9(1-16\cancel{x^2}) = 25 + 16(1-9\cancel{x^2}) - 40\sqrt{1-9x^2} \Leftrightarrow$

$\Leftrightarrow 40\sqrt{1-9x^2} = 32 \Leftrightarrow \sqrt{1-9x^2} = \frac{4}{5} \Leftrightarrow 1-9x^2 = \frac{16}{25} \Leftrightarrow 9x^2 = \frac{9}{25} \Leftrightarrow$

$\Leftrightarrow x^2 = \frac{1}{25} \Leftrightarrow x = \pm\frac{1}{5} \in \left[-\frac{1}{5}, \frac{1}{5}\right]$

Verificare: • $\arcsin(3\cdot 0) + \arcsin(4\cdot 0) = 0 = \arcsin(5\cdot 0) \Rightarrow 0$ sol

• $\arcsin\left(3\cdot\frac{1}{5}\right) + \arcsin\left(\frac{4}{5}\right) = \arcsin\frac{5}{5} \Leftrightarrow \underbrace{\arcsin\frac{3}{5}}_{\in\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]} = \underbrace{\frac{\pi}{2} - \underbrace{\arcsin\frac{4}{5}}_{\in\left[0, \frac{\pi}{2}\right]}}_{\in\left[0, \frac{\pi}{2}\right]} \Leftrightarrow$

$\Leftrightarrow \sin\left(\arcsin\frac{3}{5}\right) = \sin\left(\frac{\pi}{2} - \arcsin\frac{4}{5}\right) \Leftrightarrow$

$\Leftrightarrow \frac{3}{5} = \cos\left(\arcsin\frac{4}{5}\right) \Leftrightarrow \frac{3}{5} = \sqrt{1-\left(\frac{4}{5}\right)^2}$ (A) $\Rightarrow \frac{1}{5}$ sol

• $\arcsin\left(-\frac{3}{5}\right) + \arcsin\left(-\frac{4}{5}\right) = -\arcsin\frac{3}{5} - \arcsin\frac{4}{5} = -\arcsin\frac{5}{5} \Rightarrow -\frac{1}{5}$ sol

Așadar $S = \left\{-\frac{1}{5}, 0, \frac{1}{5}\right\}$ ■

Obs.: în locurile marcate echivalența trebuie verificată, de aici necesitatea verificării soluțiilor.

$\boxed{\text{Temă:}}$ Rezolvați $\arcsin x + \arcsin(1-x) = 0$

G. (15) Fie $A(1,0)$, $B(2,-1)$, $C(3,2)$. Calculați $\|\overline{AB} + \overline{AC}\|$

$\left.\begin{array}{l} \overline{AB} \overset{B-A}{=\!=} (2-1)\bar{i} + [(-1)-0]\bar{j} = \bar{i} - \bar{j} \\ \overline{AC} \overset{C-A}{=\!=} (3-1)\bar{i} + (2-0)\bar{j} = 2\bar{i} + 2\bar{j} \end{array}\right\} \Rightarrow (\overline{AB} + \overline{AC}) = 3\bar{i} + \bar{j} \Rightarrow \|\overline{AB} + \overline{AC}\| =$

$\Rightarrow \|\overline{AB} + \overline{AC}\| = \sqrt{3^2 + 1^2} = \sqrt{10}$ ■

$\boxed{\text{Temă:}}$ pt A, B, C de mai sus calculați $\|\overline{BC} - \overline{BA}\|$

1p 3l - Clasa a XI-a 2 15-11

A (15) Determinați A a.î. $A+\begin{pmatrix}2&1&3\\4&5&6\end{pmatrix}\cdot\begin{pmatrix}1&2&1\\0&1&2\\1&3&1\end{pmatrix}=\begin{pmatrix}1&2&3\\4&5&6\end{pmatrix}$

$$A=\begin{pmatrix}1&2&3\\4&5&6\end{pmatrix}-\begin{pmatrix}2&1&3\\4&5&6\end{pmatrix}\begin{pmatrix}1&2&1\\0&1&2\\1&3&1\end{pmatrix}=\begin{pmatrix}1&2&3\\4&5&6\end{pmatrix}-\begin{pmatrix}5&14&7\\10&31&20\end{pmatrix}=\begin{pmatrix}-4&-12&-4\\-6&-26&-14\end{pmatrix}$$

temă: Calculați AB și BA pentru $A=\begin{pmatrix}1&2&3\\4&5&6\end{pmatrix}$, $B=\begin{pmatrix}0&1\\1&0\\0&1\end{pmatrix}$

An (15) Calculați:

a) $$\lim_{n\to\infty}\sqrt{5n^2-1}-\sqrt{n+3}\overset{\boxed{\infty-\infty}}{=}\lim_{n\to\infty}\sqrt{n^2\left(5-\frac{1}{n^2}\right)}-\sqrt{n\left(1+\frac{3}{n}\right)}=$$
$$=\lim_{n\to\infty}n\left(\sqrt{5-\frac{1}{n^2}}-\frac{1}{\sqrt{n}}\sqrt{1-\frac{3}{n}}\right)\overset{\infty\cdot\sqrt{5}}{=}\infty$$

b) $$\lim_{n\to\infty}\sqrt{n^3-4n}-\sqrt{5n^3-4}\overset{\boxed{\infty-\infty}}{=}\lim_{n\to\infty}\sqrt{n^3}\left[\sqrt{1-\frac{4}{n^2}}-\sqrt{5-\frac{4}{n^3}}\right]\overset{\infty\cdot(1-\sqrt{5})}{=}-\infty$$

c) $$\lim_{n\to\infty}\sqrt{4n^2+5n-6}-2n\overset{\boxed{\infty-\infty}}{=}\lim_{n\to\infty}\frac{4n^2+5n-6-4n^2}{\sqrt{4n^2+5n-6}+2n}=\lim_{n\to\infty}\frac{5n-6}{n\left[\sqrt{4+\frac{5}{n}-\frac{6}{n^2}}+2\right]}=$$
$$=\lim_{n\to\infty}\frac{n\left(5-\frac{6}{n}\right)}{n\left[\sqrt{4+\frac{5}{n}-\frac{6}{n^2}}+2\right]}=\frac{5}{4}$$

d) $$\lim_{n\to\infty}\sqrt{9n^2-5n+1}-\sqrt{4n^2+1}-n=\lim_{n\to\infty}\sqrt{9n^2-5n+1}-3n-\left(\sqrt{4n^2+1}-2n\right)=$$
$$=\underbrace{\lim_{n\to\infty}\sqrt{9n^2-5n+1}-3n}_{=-\frac{5}{6}\ \boxed{\text{temă}}}-\underbrace{\lim_{n\to\infty}\sqrt{4n^2+1}-2n}_{=0\ \boxed{\text{temă}}}=-\frac{5}{6}-0=-\frac{5}{6}$$

$\boxed{\text{temă}}$: Determinați $a\in\mathbb{R}$ a.î. $\lim_{n\to\infty}\sqrt{4n^2+an+3}-2n$ să fie:

a) 0; b) 7; c)* ∞