1p 22 - Clasa a IX-a   Z 14 - 09

A. (14) Să se arate că $a = \underbrace{11\ldots1}_{n\text{ cifre}}\underbrace{55\ldots5}_{n-1\text{ cifre}}6$ e pătrat perfect

$$a = 11\ldots1 \cdot 10^n + 55\ldots5 \cdot 10 + 6 =$$

$$55\ldots5 = 5(1+10+\ldots+10^{n-2}) = 5 \cdot 1 \cdot \frac{10^{n-1}-1}{9}$$

$$11\ldots1 = 1+10+10^2+\ldots+10^{n-1} = 1 \cdot \frac{10^n-1}{9}$$

$$\Rightarrow a = \frac{10^n-1}{9}\cdot 10^n + 5\cdot\frac{10^n-10}{9} + \overset{9)}{6} =$$

$$= \frac{10^{2n}-10^n+5\cdot 10^n-50+54}{9} = \frac{10^{2n}+4\cdot 10^n+4}{9} = \frac{(10^n+2)^2}{9} \Rightarrow$$

$$\Rightarrow a = \left(\frac{10^n+2}{3}\right)^2$$

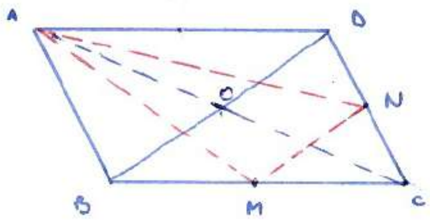
Suma cifrelor numărului 10^n+2 e $1+2=3 \,\vdots\, 3 \Rightarrow (10^n+2) \,\vdots\, 3 \Rightarrow \frac{10^n+2}{3} = b \in \mathbb{N}$ $\Rightarrow$

$\Rightarrow a = b^2$ pătrat perfect. ∎

Temă: Arătați că $A = \underbrace{99\ldots9}_{n\text{ cifre}}82\underbrace{00\ldots0}_{n\text{ cifre}}81$ este pătrat perfect.

G. (14)

I	ABCD paralelogram
	$AC \cap BD = \{O\}$
	$\overline{BM} + \overline{CM} = \overline{0}$
	$\overline{CN} + \overline{DN} = \overline{0}$
C	O cdg al $\triangle AMN$



$\overline{BM} + \overline{CM} = \overline{0} \Rightarrow \overline{BM} = \overline{MC} \Rightarrow M$ mijloc $[BC]$

$\overline{CN} + \overline{DN} = \overline{0} \Rightarrow \frac{\overline{NC}}{\overline{ND}} = -1 \Rightarrow N$ mijloc $[CD]$

fie Q cdg al $\triangle AMN \Rightarrow \overline{AQ} = \frac{1}{3}(\overline{AA} + \overline{AM} + \overline{AN}) = \frac{1}{3}(\overline{AM} + \overline{AN})$

$\overline{AM} + \overline{AN} = \frac{1}{2}(\overline{AB} + \overline{AC}) + \frac{1}{2}(\overline{AC} + \overline{AD}) = \frac{1}{2}(\overline{AB} + \overline{AD}) + \overline{AC} = \frac{3}{2}\overline{AC}$ $\Rightarrow$

$\Rightarrow \overline{AQ} = \frac{1}{3}\cdot\frac{3}{2}\overline{AC} = \frac{1}{2}\overline{AC} = \overline{AO} \Rightarrow Q = O$ ∎

Am folosit: Th. de caract. a mijlocului unui segment:

fie $M \in AB$, $(A \neq B)$. UASE: 1) M mijl. $[AB]$
2) $\forall P$ $\overline{PM} = \frac{1}{2}(\overline{PA} + \overline{PB})$
3) $\exists P$ a.î. $\overline{PM} = \frac{1}{2}(\overline{PA} + \overline{PB})$

Temă: Aplicați Teorema Sud America pt a demonstra Th. de caract a mijlocului unui segment

T.(14) Demonstrați că $\operatorname{arctg} x + \operatorname{arcctg} x = \frac{\pi}{2} \quad \forall x \in \mathbb{R}$.

Fie $x \in \mathbb{R}$

$$\underbrace{\operatorname{arctg} x}_{\in\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)} = \underbrace{\frac{\pi}{2} - \underbrace{\operatorname{arcctg} x}_{\in (0,\pi)}}_{\in\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)} \underset{\text{pe } \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)}{\overset{\text{tg bij}}{\Longleftrightarrow}} \operatorname{tg}(\operatorname{arctg} x) = \operatorname{tg}\left(\frac{\pi}{2} - \operatorname{arcctg} x\right) \Leftrightarrow$$

$$\Leftrightarrow x = \operatorname{ctg}(\operatorname{arcctg} x) \quad (A) \quad \blacksquare$$

$\boxed{\text{Temă:}}$ $\arcsin x + \arccos x = \frac{\pi}{2} \quad \forall x \in [-1, 1]$

G (14) Fie $A(1,2)$, $B(2,0)$, $C(0,3)$. Determinați câte un versor al vectorilor $\overline{AB}$ și $\overline{AC}$

$$\overline{AB} \overset{B-A}{=\!=} (2-1)\bar{i} + (0-2)\bar{j} = \bar{i} - 2\bar{j} \Rightarrow |\overline{AB}| = \sqrt{1^2 + (-2)^2} = \sqrt{5} \Rightarrow \text{aleg } \bar{u} = \frac{1}{\sqrt{5}}(\bar{i} - 2\bar{j}) = \frac{1}{\sqrt{5}}\bar{i} - \frac{2}{\sqrt{5}}\bar{j}$$

$$\overline{AC} \overset{C-A}{=\!=} (0-1)\bar{i} + (3-2)\bar{j} = -\bar{i} + \bar{j} \Rightarrow |\overline{AC}| = \sqrt{2} \Rightarrow \text{aleg } \bar{v} = \frac{1}{\sqrt{2}}(-\bar{i} + \bar{j}) = -\frac{1}{\sqrt{2}}\bar{i} + \frac{1}{\sqrt{2}}\bar{j} \quad \blacksquare$$

Al (14) Rezolvați:
$$\begin{cases} A + 2B^t = \begin{pmatrix} 5 & 2 \\ 4 & 5 \end{pmatrix} & (*) \\ A^t - B = \begin{pmatrix} -1 & 1 \\ -1 & 2 \end{pmatrix} & (**) \end{cases}$$

$$\begin{cases} A + 2B^t = \begin{pmatrix} 5 & 2 \\ 4 & 5 \end{pmatrix} \\ (A^t - B)^t = \begin{pmatrix} -1 & -1 \\ 1 & 2 \end{pmatrix} \end{cases} \Leftrightarrow \begin{cases} A + 2B^t = \begin{pmatrix} 5 & 2 \\ 4 & 5 \end{pmatrix} \\ A - B^t = \begin{pmatrix} -1 & -1 \\ 1 & 2 \end{pmatrix} \end{cases} \Leftrightarrow \begin{cases} A = \begin{pmatrix} 1 & 0 \\ 2 & 3 \end{pmatrix} \\ B^t = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} \end{cases} \Leftrightarrow \begin{cases} A = \begin{pmatrix} 1 & 0 \\ 2 & 3 \end{pmatrix} \\ B = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} \end{cases}$$

$\boxed{\text{Temă}}$:
$$\begin{cases} 2A + B^t = \begin{pmatrix} 4 & 1 & 2 \\ 5 & 7 & 6 \end{pmatrix} \\ A^t + B = \begin{pmatrix} 3 & 3 \\ 1 & 4 \\ 1 & 4 \end{pmatrix} \end{cases}$$

An (14) Calculați:

a) $$\lim_{n\to\infty} \frac{2n^2-1}{3n-n^2} \overset{\boxed{\frac{\infty}{-\infty}}}{=} \lim_{n\to\infty} \frac{\cancel{n^2}\left(2-\frac{1}{n}\right)}{\cancel{n^2}\left(\frac{3}{n}-1\right)} = \frac{2}{-1} = -2$$

b) $$\lim_{n\to\infty} \frac{n-1}{6n^3+5n^2+6} \overset{\boxed{\frac{\infty}{\infty}}}{=} \lim_{n\to\infty} \frac{\overset{1}{\cancel{n}}\left(1-\frac{1}{n^3}\right)}{\underset{n^2}{n^{\cancel{3}}}\left(6+\frac{5}{n}+\frac{6}{n^3}\right)} \overset{\boxed{\frac{1}{\infty\cdot 6}}}{=} 0$$

c) $$\lim_{n\to\infty} \frac{n^6-1}{n^3+n+1} \overset{\boxed{\frac{\infty}{\infty}}}{=} \lim_{n\to\infty} \frac{n^6\left(1-\frac{1}{n^6}\right)}{n^3\left(1+\frac{1}{n^2}+\frac{1}{n^3}\right)} = \infty$$

Obs: De obicei redactăm "rapid" astfel:

$$\lim_{n\to\infty} \frac{2n^2-1}{3n-n^2} = \lim_{n\to\infty} \frac{2n^2}{-n^2} = -2$$

$$\lim_{n\to\infty} \frac{n-1}{6n^5+5n^2+6} = \lim_{n\to\infty} \frac{n}{6n^5} = 0$$

$$\lim_{n\to\infty} \frac{n^6-1}{n^3+n+1} = \lim_{n\to\infty} \frac{n^6}{n^3} = \infty$$

$\boxed{\text{Temă}}$: Calculați:

a) $\lim_{n\to\infty} \frac{n^5+n+1}{2n^2-2}$

b) $\lim_{n\to\infty} \frac{n^5+n-1}{3n-4n^5}$

c) $\lim_{n\to\infty} \frac{n^2+1}{5n^3-1}$

d) $\lim_{n\to-\infty} \frac{6n^5+1}{1-5n^2}$