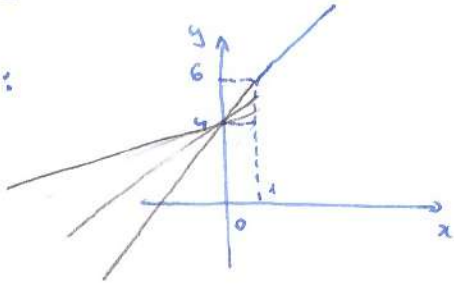
A. (5) Fie $f: \mathbb{R} \to \mathbb{R}$, $f(x) = \begin{cases} ax+4, & x \le 1 \\ x+5, & x > 1 \end{cases}$. Det $a \in \mathbb{R}$ pt care $f \nearrow$.

Ls $\begin{cases} a > 0 \\ a \cdot 1 + 4 \le 1+5 \end{cases} \Leftrightarrow \begin{cases} a > 0 \\ a \le 2 \end{cases} \Rightarrow a \in (0, 2]$ ■

Într-adevăr:



$\boxed{\text{temă}}$: pt ce valoare a lui a $\operatorname{Im} f = \mathbb{R}$?

T. (5) Demonstrați că $\alpha = \dfrac{\sin^6 x + \cos^6 x + \frac{3}{4}\sin^2 2x}{\sin^4 x + \cos^4 x + \frac{1}{2}\sin^2 2x}$ nu depinde de x

$1^3 = (\sin^2 x + \cos^2 x)^3 = \sin^6 x + \cos^6 x + 3\sin^2 x \cos^2 x(\sin^2 x + \cos^2 x) =$

$= \sin^6 x + \cos^6 x + \frac{3}{4}\cdot(2\sin x \cos x)^2 \cdot 1 =$

$= \sin^6 x + \cos^6 x + \frac{3}{4} \cdot \sin 2x$

$1^2 = (\sin^2 x + \cos^2 x)^2 = \sin^4 x + \cos^4 x + 2\sin^2 x \cos^2 x =$

$= \sin^4 x + \cos^4 x + \frac{1}{2}\cdot(2\sin x \cos x)^2 =$

$= \sin^4 x + \cos^4 x + \frac{1}{2} \cdot \sin 2x$

Așadar $\alpha = \dfrac{1^3}{1^2} = 1$ ■

$\boxed{\text{temă}}$: Demonstrați că $E = \cos\left(\frac{\pi}{3} - x\right)\cos x - \sin x \sin\left(\frac{\pi}{3} - x\right)$ nu depinde de x.

1. Se consideră x_1 și x_2 soluțiile ecuației $x^2-2(m+1)x+m+1=0$, unde m este număr real.

a) Pentru $m=0$, rezolvați ecuația.

b) Determinaţi numerele reale m pentru care o soluție a ecuației este dublul celeilalte soluții.

c) Determinaţi valorile reale ale lui m pentru care $x_1^2+x_2^2-x_1x_2(x_1+x_2)>0$.

a) $x^2-2x+1=0$	**3p**
$x_1=x_2=1$	**2p**
b) $x_1+x_2=2(m+1)$, $x_1x_2=m+1 \Rightarrow x_1+x_2=2x_1x_2$	**2p**
$x_1=2x_2 \Leftrightarrow 3x_2=4x_2^2 \Leftrightarrow x_2=0$ sau $x_2=\frac{3}{4}$, de unde obținem $m=-1$ sau $m=\frac{1}{8}$	**3p**
c) $x_1^2+x_2^2-x_1x_2(x_1+x_2)=4(m+1)^2-2(m+1)-2(m+1)^2=2m(m+1)$	**3p**
$2m(m+1)>0 \Leftrightarrow m\in(-\infty,-1)\cup(0,+\infty)$	**2p**

T 09 Arătați că $\operatorname{arctg}\frac{1}{3}+\operatorname{arctg}\frac{1}{5}+\operatorname{arctg}\frac{1}{7}+\operatorname{arctg}\frac{1}{8}=\frac{\pi}{4}$

$\frac{1}{8}<\frac{1}{7}<\frac{1}{5}<\frac{1}{3}<\frac{1}{\sqrt{3}}$ | $\Rightarrow$ $0<\operatorname{arctg}\frac{1}{8}<\operatorname{arctg}\frac{1}{7}<\operatorname{arctg}\frac{1}{5}<\operatorname{arctg}\frac{1}{3}<\operatorname{arctg}\frac{1}{\sqrt{3}}=\frac{\pi}{6}$

arctg $\nearrow$ pe $\mathbb{R}$ |

Rescriem egalitatea $\underbrace{\operatorname{arctg}\frac{1}{3}+\operatorname{arctg}\frac{1}{5}+\operatorname{arctg}\frac{1}{7}}_{\in\left(0,\frac{\pi}{2}\right)}=\underbrace{\frac{\pi}{4}-\operatorname{arctg}\frac{1}{8}}_{\in\left(0,\frac{\pi}{4}\right)}$

tg e inj pe $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ $\Rightarrow$

$\Rightarrow$ egalitatea este echivalentă cu

$$\operatorname{tg}\left(\operatorname{arctg}\frac{1}{3}+\operatorname{arctg}\frac{1}{5}+\operatorname{arctg}\frac{1}{7}\right)=\operatorname{tg}\left(\frac{\pi}{4}-\operatorname{arctg}\frac{1}{8}\right)$$

$$MD=\operatorname{tg}\left(\frac{\pi}{4}-\operatorname{arctg}\frac{1}{8}\right)=\frac{\operatorname{tg}\frac{\pi}{4}-\operatorname{tg}\left(\operatorname{arctg}\frac{1}{8}\right)}{1+\operatorname{tg}\frac{\pi}{4}\cdot\operatorname{tg}\operatorname{arctg}\frac{1}{8}}=\frac{1-\frac{1}{8}}{1+1\cdot\frac{1}{8}}=\frac{7}{9}$$

$$MS=\operatorname{tg}\left(\operatorname{arctg}\frac{1}{3}+\operatorname{arctg}\frac{1}{5}+\operatorname{arctg}\frac{1}{7}\right)=\frac{\operatorname{tg}\operatorname{arctg}\frac{1}{3}+\operatorname{tg}\left(\operatorname{arctg}\frac{1}{5}+\operatorname{arctg}\frac{1}{7}\right)}{1-\operatorname{tg}\operatorname{arctg}\frac{1}{3}\cdot\operatorname{tg}\left(\operatorname{arctg}\frac{1}{5}+\operatorname{arctg}\frac{1}{7}\right)}$$

Cum $\operatorname{tg}\left(\operatorname{arctg}\frac{1}{5}+\operatorname{arctg}\frac{1}{7}\right)=\frac{\frac{1}{5}+\frac{1}{7}}{1-\frac{1}{5}\cdot\frac{1}{7}}=\frac{12}{34}=\frac{6}{17}$ $\Rightarrow$

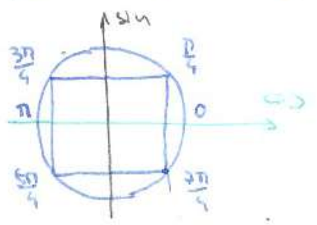
$$\Rightarrow MS=\frac{\frac{1}{3}+\frac{6}{17}}{1-\frac{1}{3}\cdot\frac{6}{17}}=\frac{35}{51-6}=\frac{35}{45}=\frac{7}{9}=MD \quad \text{cctd}$$

$\boxed{\text{Temă:}}$ Calculați $\operatorname{arctg}\frac{1}{2}+\operatorname{arctg}\frac{1}{5}+\operatorname{arctg}\frac{1}{8}$ și compuneți un exercițiu asemănător celui de mai sus!

A 09 Scrieți sub f. trig.:

$z=-1+i$

$|z|=\sqrt{2}\Rightarrow z=\sqrt{2}\left(-\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}i\right)=\sqrt{2}\left(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right)$



$\boxed{\text{Temă}}$: La fel pt $1-i$, $-1-i$, $1+\sqrt{3}i$, $1-\sqrt{3}i$, $-1+\sqrt{3}i$, $-1-\sqrt{3}i$

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A 09 Fie $A(x)=\begin{pmatrix}1-x & 0 & x\\ 0 & 0 & 0\\ x & 0 & 1-x\end{pmatrix}$, $x\in\mathbb{R}$

a) Rezolvați ecuația $\det A(x) = 0$

b) Demonstrați că $A(x)\cdot A(y) = A(x+y-2xy)$ $\forall x,y\in\mathbb{R}$

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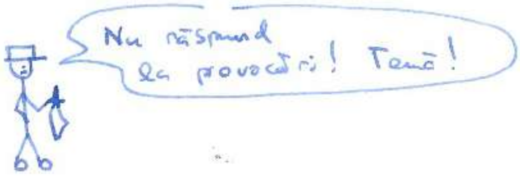
a) $\det A(x) = \begin{vmatrix}1-x & 0 & x\\ 0 & 0 & 0\\ x & 0 & 1-x\end{vmatrix} \xlongequal[\text{e nulă}]{\text{linia 2}} 0 \quad \forall x \Rightarrow S=\mathbb{R}$

b)



An 09 Fie $f:\mathbb{R}\to\mathbb{R}$, $f(x)=\sqrt{x^2-x+1}$

a) calculați $\lim\limits_{x\to\infty}\left(\frac{f(x)}{x}\right)^x$

b) Arătați că $f'(x)=\frac{2x-1}{2\sqrt{x^2-x+1}}$ și propuneți un exercițiu mai inteligent care să se bazeze pe această

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a) $\lim\limits_{x\to\infty}\frac{f(x)}{x} = \lim\limits_{x\to\infty}\frac{\sqrt{x^2-x+1}}{x} = \lim\limits_{x\to\infty}\frac{|x|\sqrt{1-\frac{1}{x}+\frac{1}{x^2}}}{x} = 1 \Rightarrow$ cazul $\boxed{1^\infty}$

$$\lim_{x\to\infty}\left(\frac{f(x)}{x}\right)^x = \lim_{x\to\infty}\left(1+\frac{f(x)}{x}-1\right)^x = \lim_{x\to\infty}\Bigg[\underbrace{\left(1+\frac{\sqrt{x^2-x+1}-x}{x}\right)^{\frac{x}{\sqrt{x^2-x+1}-x}}}_{\to e}\Bigg]^{\frac{\sqrt{x^2-x+1}-x}{x}\cdot x} =$$

$= e^{l}$ unde

$$l = \lim_{x\to\infty}(\sqrt{x^2-x+1}-x) \overset{\text{conj}}{=} \lim_{x\to\infty}\frac{\cancel{x^2}-x+1-\cancel{x^2}}{\sqrt{x^2-x+1}+x} = \lim_{x\to\infty}\frac{x\left(-1+\frac{1}{x}\right)}{x\left(\sqrt{1-\frac{1}{x}+\frac{1}{x^2}}+1\right)} = -\frac{1}{2}$$

$\Rightarrow$ limita e $e^{-\frac{1}{2}} = \frac{1}{\sqrt{e}}$

b)