1 PZ - clasa a IX - a

A ① Rez. inecuația

$$|x-5| \le 10-x$$

(I) Dacă $10-x<0 \ (\Leftrightarrow x>10)$ inecuația nu are soluții

(II) Dacă $10-x=0$ inecuația nu se verifică

(III) Dacă $10-x>0$ inecuația devine

$$-(10-x) \le x-5 \le 10-x \Leftrightarrow \begin{cases} x-10 \le x-5 \\ x-5 \le 10-x \end{cases} \Leftrightarrow \begin{cases} -10 \le -5 \ (A) \ \forall x \\ 2x \le 15 \Leftrightarrow x \le \frac{15}{2} \end{cases}$$

Cum $10-x>0 \Rightarrow x<10$ $\Rightarrow$

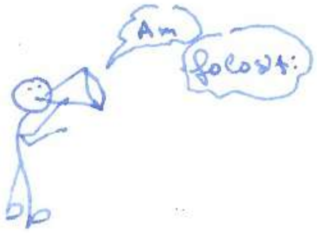
$$\Rightarrow S=\left(-\infty, \frac{15}{2}\right]$$

temă: Rez ecuația $|3-3x| = |2x+8|$

T ① Să se simplifice expresia

$$E = \frac{\sin x+\sin y-\sin(x+y)}{\sin x+\sin y+\sin(x+y)} = \frac{2\sin\frac{x+y}{2}\cos\frac{x-y}{2} - 2\sin\frac{x+y}{2}\cos\frac{x+y}{2}}{2\sin\frac{x+y}{2}\cos\frac{x-y}{2} + 2\sin\frac{x+y}{2}\cos\frac{x+y}{2}} =$$

$$= \frac{2\sin\frac{x+y}{2}\left(\cos\frac{x-y}{2} - \cos\frac{x+y}{2}\right)}{2\sin\frac{x+y}{2}\left(\cos\frac{x-y}{2} + \cos\frac{x+y}{2}\right)} = \frac{-2\sin\frac{x}{2}\sin\frac{-y}{2}}{2\cos\frac{x}{2}\cos\frac{y}{2}} = 2\,tg\frac{x}{2}\,tg\frac{y}{2}$$



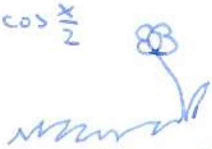
$$\sin x+\sin y = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2}$$

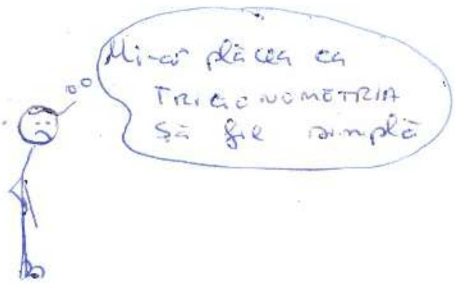
$$\cos x+\cos y = 2\cos\frac{x+y}{2}\cos\frac{x-y}{2}$$

$$\cos x-\cos y = -2\sin\frac{x-y}{2}\sin\frac{x+y}{2}$$

$$\sin x = 2\sin\frac{x}{2}\cos\frac{x}{2}$$

$$\frac{\sin x}{\cos x} = tg\, x$$





OK: Simplificați:

$$\frac{\sin(x-y)-\sin x+\sin y}{\sin(x-y)+\sin x-\sin y}$$

temă

Clasa a X-a Z 01-10

A ① Rezolvați ecuația:

$$\log_3 x + \log_{\sqrt{3}} x + \log_{\frac{1}{3}} x = 6$$

CE: $x > 0$

$\log_3 x \overset{\text{not}}{=} \alpha \Rightarrow \log_{\sqrt{3}} x = \log_{3^{\frac{1}{2}}} x = \frac{1}{\frac{1}{2}} \log_3 x = 2\log_3 x = 2\alpha$

$\log_{\frac{1}{3}} x = \log_{3^{-1}} x = -\log_3 x = -\alpha$ $\Rightarrow$

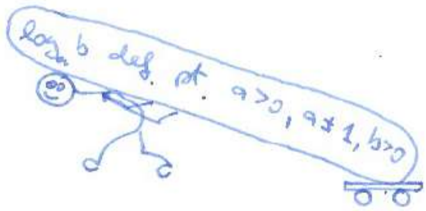
$\Rightarrow \alpha + 2\alpha - \alpha = 6 \Rightarrow \alpha = 3 \Rightarrow \log_3 x = 3 \Rightarrow \boxed{x = 3^3} > 0$ soluție

Temă: Rez ecuația: $\log_3 x + \log_9 x + \log_{27} x = \frac{11}{2}$

Am folosit:

$$\log_{a^m} b^n = \frac{n}{m} \log_a b$$

$$\log_a b + \log_a c = \log_a bc$$



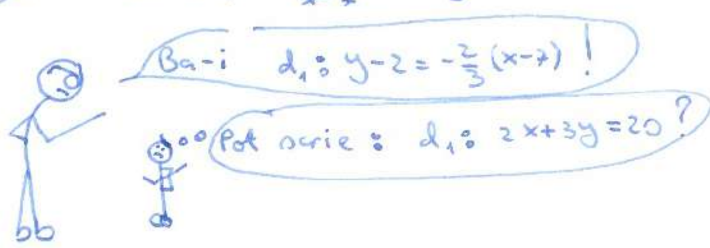
G. ① Scrieți ecuația paralelei prin A(7,2) la d: $2x+3y=6$

$d: 3y = 6 - 2x \Rightarrow d: y = -\frac{2}{3}x + 6 \Rightarrow m_d = -\frac{2}{3}$

fie $d_1 \parallel d \Rightarrow m_{d_1} = m_d = -\frac{2}{3}$

$d_1 \ni A(7,2)$

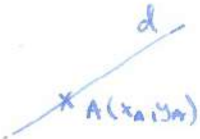
$\Rightarrow d_1: \frac{y-2}{x-7} = -\frac{2}{3}$ CNN



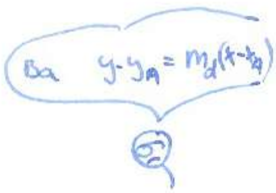
Am folosit:

• pt $d: y = mx + n \Rightarrow m = m_d$

• pt



$\Rightarrow d: \frac{y-y_A}{x-x_A} = m_d$ CNN



temă: Scrieți ecuația dreptei ce trece prin A(1,-2) și e paralelă cu dreapta CD știind că C(2,-1), D(-4,5)

1pz – clasa a $\overline{XI}$-a Z 01 – 11

A ① Fie $f:(0,\infty) \longrightarrow \mathbb{R}$, $f(x) = \ln x - \frac{2(x-1)}{x+1}$

a) Calculați $f'(x)$ pt $x \in D_{f'}$

b) Determinați punctele în care tangenta la grafic e paralelă cu $d: 9y = 2x$

c) Arătați că pt $x > 1$ $\ln x \geq \frac{2(x-1)}{x+1}$

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a) $f'(x) = \frac{1}{x} - 2 \cdot \frac{x+1-(x-1)}{(x+1)^2} = \frac{1}{x} - 2 \cdot \frac{2}{(x+1)^2} = \frac{(x+1)^2 - 4x}{x(x+1)^2} = \frac{(x-1)^2}{x(x+1)^2} \quad \forall x \in (0,\infty)$

b) $d: y = \frac{2}{9}x \Rightarrow m_d = \frac{2}{9}$

fie $A(x_A, f(x_A)) \in G_f \Rightarrow m_{t_A} = f'(x_A) = \frac{(x_A-1)^2}{x_A(x_A+1)^2}$

$t_A \parallel d \Rightarrow m_{t_A} = m_d$ $\Bigg| \Rightarrow$

$\Rightarrow x_A$ verifică ecuația $\frac{(x-1)^2}{x(x+1)^2} = \frac{2}{9} \Leftrightarrow 9x^2 - 18x + 9 = 2x^3 + 4x^2 + 2x$

$\Leftrightarrow 2x^3 - 5x^2 + 20x - 9 = 0 \Rightarrow$ folosim Schema lui Horner:

	3	2	1	0
	2	-5	20	-9
$\frac{1}{2}$	2	-4	18	0

$\Rightarrow 2x^3 - 5x^2 + 20x - 9 = \left(x - \frac{1}{2}\right)\underbrace{\left(2x^2 - 4x + 18\right)}_{\Delta < 0} \Rightarrow$

$\Rightarrow x = \frac{1}{2} > 0$ unica soluție $\Rightarrow$

$\Rightarrow$ Răspuns: $A\left(\frac{1}{2}, f\left(\frac{1}{2}\right)\right)$ adică $A\left(\frac{1}{2}, -\ln 2 + \frac{2}{3}\right)$

c) $\ln x \geq \frac{2(x-1)}{x+1} \Leftrightarrow \ln x - \frac{2(x-1)}{x+1} \geq 0 \Leftrightarrow f(x) \geq 0$.

x		0		1	
$f'(x) = \frac{(x-1)^2}{x(x+1)^2}$	////	(	+		+
$f(x)$	////	(	↗	0	↗
$f(x)$	////	(	−	0	+

$\Rightarrow f(x) \geq f(1) \quad \forall x > 1 \Rightarrow$

$\Rightarrow f(x) \geq 0 \quad \forall x > 1$