Secțiunea 20 – Sisteme de ecuații liniare

Exersare Of: 10p

1. $\Delta_s = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 4 & -1 & 1 \end{vmatrix} \overset{C_3 C_2}{=} \begin{vmatrix} 1 & 1 & 2 \\ 2 & 1 & 0 \\ 4 & -1 & 0 \end{vmatrix} = 2\begin{vmatrix} 2 & 1 \\ 4 & -1 \end{vmatrix} = -12 \neq 0 \overset{Th}{\underset{Cramer}{\Longrightarrow}}$ sist. compatibil determinat 2p 2p

$\Delta_x = \begin{vmatrix} 3 & 1 & 1 \\ 2 & 1 & -1 \\ 4 & -1 & 1 \end{vmatrix} \overset{C_3-C_2}{=} \begin{vmatrix} 3 & 1 & 2 \\ 2 & 1 & 0 \\ 4 & -1 & 0 \end{vmatrix} = -12 \Rightarrow x = \frac{\Delta_x}{\Delta_s} = 1$ 2p

$\Delta_y = \begin{vmatrix} 1 & 3 & 1 \\ 2 & 2 & -1 \\ 4 & 4 & 1 \end{vmatrix} \overset{C_2-C_1}{=} \begin{vmatrix} 1 & 2 & 1 \\ 2 & 0 & -1 \\ 4 & 0 & 1 \end{vmatrix} = -2\begin{vmatrix} 2 & -1 \\ 4 & 1 \end{vmatrix} = -12 \Rightarrow y = \frac{\Delta_y}{\Delta_s} = 1$ 2p

Cum $x+y+z=3 \Rightarrow 1+1+z=3 \Rightarrow z=1$ 2p $\Rightarrow S=\{(1,1,1)\}$

2. $\begin{cases} x+y+z=1 \\ x+2y+2z=-1 \\ x-y+2z=2 \end{cases} \underset{l_3-l_1}{\overset{l_2-l_1}{\Longleftrightarrow}} \begin{cases} x+y+z=1 \\ y+z=-2 \\ -2y+z=1 \end{cases} \overset{l_3+2l_2}{\Longleftrightarrow} \begin{cases} x+y+z=1 \\ y+z=-2 \\ 3z=-3 \end{cases} \Leftrightarrow \begin{cases} x=3 \\ y=-1 \\ z=-1 \end{cases} \Leftrightarrow S=\{(3,-1,-1)\}$ 10p

3. a) $\bar{A} = \left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 1 & 2 & 3 & 1 \\ 1 & a^2 & a & 1 \end{array}\right)$

$\Delta_s = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & a^2 & a \end{vmatrix} \underset{C_3-C_1}{\overset{C_2-C_1}{=}} \begin{vmatrix} 1 & 0 & 0 \\ 1 & 1 & 2 \\ 1 & a^2-1 & a-1 \end{vmatrix} = \begin{vmatrix} 1 & 2 \\ a^2-1 & a-1 \end{vmatrix} = (a-1)-2(a^2-1) = -[2a^2-a-1]$ 4p

Sist. este compatibil determinat $\underset{2p}{\overset{\text{Cramer in } \mathbb{R}}{\Longleftrightarrow}} \Delta_s \neq 0 \Leftrightarrow 2a^2-a-1 \neq 0 < \begin{matrix} 1 \\ -\frac{1}{2} \end{matrix} \overset{2p}{\Rightarrow}$

$\Rightarrow a \in \mathbb{R} \setminus \{-\frac{1}{2}, 1\}$ 2p

b) pt $a=-8$ sistemul este compatibil determinat și $\Delta_s = -[2\cdot 8^2-8-1] = -119$ 2p

Cum $\bar{A} = \left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 1 & 2 & 3 & 1 \\ 1 & 64 & 8 & 1 \end{array}\right)$ obținem $\Delta_x = \begin{vmatrix} 2 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 64 & 8 \end{vmatrix} = -295 \Rightarrow x = \frac{295}{119}$ 2p

$\Delta_y = \begin{vmatrix} 1 & 2 & 1 \\ 1 & 1 & 3 \\ 1 & 1 & 8 \end{vmatrix} \overset{C_2-C_1}{=} \begin{vmatrix} 1 & 1 & 1 \\ 1 & 0 & 3 \\ 1 & 0 & 8 \end{vmatrix} = -\begin{vmatrix} 1 & 3 \\ 1 & 8 \end{vmatrix} = -5 \Rightarrow y = \frac{5}{119}$ 2p

$\Delta_z = \begin{vmatrix} 1 & 1 & 2 \\ 1 & 2 & 1 \\ 1 & 64 & 1 \end{vmatrix} \overset{C_3-C_1}{=} \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 0 \\ 1 & 64 & 0 \end{vmatrix} = 62 \Rightarrow z = \frac{-62}{119}$ 2p $\Rightarrow S=\{(\frac{295}{119}, \frac{5}{119}, \frac{-62}{119})\}$ 2p

c) pt $a=1 \Rightarrow \Delta_s = 0$ 2p și $\bar{A} = \left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 1 & 2 & 3 & 1 \\ 1 & 1 & 1 & 1 \end{array}\right)$

Cum sistemul devine $\begin{cases} x+y+z=2 \\ \text{ecuația 2 nu contează} \\ x+y+z=1 \end{cases} \Rightarrow$ ecuațiile 1 și 3

nu se verifică simultan $\Rightarrow S=\emptyset$ și sistemul este incompatibil

8p

Secțiunea 20 - Sisteme de ecuații liniare.

Aprofundare Of: 10p

1. a) pt $m=0$ $\quad \det A = \begin{vmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 2 & 0 \end{vmatrix} \overset{l_3-l_2}{=} \begin{vmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{vmatrix} = -\begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix} = 3$ 10p

b) A singulară $\Leftrightarrow \det A = 0$. 2p

$$\det A = \begin{vmatrix} 2m & 1 & 1 \\ 1 & 2m & 1 \\ 1 & 2 & 2m \end{vmatrix} \overset{\substack{C_1-2mC_3 \\ C_2-C_3}}{=} \begin{vmatrix} 0 & 0 & 1 \\ 1-2m & 2m-1 & 1 \\ 1-4m^2 & 2-2m & 2m \end{vmatrix} = \begin{vmatrix} 1-2m & 2m-1 \\ 1-4m^2 & 2-2m \end{vmatrix} =$$

$$= (1-2m)\begin{vmatrix} 1 & -1 \\ 1-4m^2 & 2-2m \end{vmatrix} = (1-2m)(2-2m+1-4m^2) = (2m-1)(4m^2+2m-3)$$ 4p

Așadar $\det A = 0 \Leftrightarrow m = \frac{1}{2}$ 2p sau $4m^2+2m-3=0 \Leftrightarrow m = \frac{-1\pm\sqrt{13}}{4}$ 2p

$\Rightarrow S = \left\{ \frac{1}{2}, \frac{-1-\sqrt{13}}{4}, \frac{-1+\sqrt{13}}{4} \right\}$

c) pt $m=-1 \Rightarrow$ sistemul devine $\begin{cases} -2x+y+z=-1 \\ x-2y+z=0 \\ x+2y-2z=1 \end{cases}$ 2p $\Rightarrow S = \left\{ \left(\frac{1}{3}, 0, -\frac{1}{3}\right) \right\}$ 6p $\Rightarrow$

$\Rightarrow$ cel mult unul dintre numerele a, b, c este întreg (doar $0 \in \mathbb{Z}$) 2p

2. a) $\bar{A} = \left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ a+1 & -1 & 1 & 0 \\ 1 & 1 & -a & 1 \end{array} \right)$

$$\det A = \begin{vmatrix} 1 & 1 & 1 \\ a+1 & -1 & 1 \\ 1 & 1 & -a \end{vmatrix} \overset{l_3-l_1}{=} \begin{vmatrix} 1 & 1 & 1 \\ a+1 & -1 & 1 \\ 0 & 0 & -a-1 \end{vmatrix} = (-a-1)\begin{vmatrix} 1 & 1 \\ a+1 & -1 \end{vmatrix} = (a+1)(a+2)$$

pt $a=-1 \Rightarrow \det A = 0 \Rightarrow$ A singulară 10p

b) A singulară $\overset{2p}{\Leftrightarrow} \det A = 0 \overset{6p}{\Leftrightarrow} (a+1)(a+2)=0 \overset{2p}{\Leftrightarrow} a \in \{-1, -2\}$

c) Sistemul are soluție unică în $\mathbb{R} \underset{2p}{\overset{\text{Cramer}}{\Longleftrightarrow}} \Delta_s \neq 0 \overset{2p}{\Leftrightarrow} a \in \mathbb{R} \setminus \{-1, -2\}$

Cum $S = \left\{ \left(\frac{2a}{a^2+3a+2}, \frac{2a^2+3a+2}{a^2+3a+2}, \frac{1}{a+1} \right) \right\}$ 2p obținem

$2x_0 + y_0 z_0 = 0 \Leftrightarrow 2\cdot\frac{2a}{a^2+3a+2} + \frac{2a^2+3a+2}{(a^2+3a+2)(a+1)} = 0 \Leftrightarrow 4a(a+1)+2a^2+3a+2=0$

$\Leftrightarrow 6a^2+7a+2=0 \overset{2p}{\Leftrightarrow} a \in \left\{-\frac{2}{3}; -\frac{1}{2}\right\} \subset \mathbb{R}\setminus\{-1,-2\} \Rightarrow S \overset{2p}{=} \left\{-\frac{2}{3}, -\frac{1}{2}\right\}$

3. a) pt $a=9$ $\quad \det A = \begin{vmatrix} 1 & 1 & 2 \\ 1 & 2 & 9 \\ -2 & -1 & 3 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 2 \\ 0 & 1 & 7 \\ 0 & 1 & 7 \end{vmatrix} = 0 \Rightarrow$ A singulară 10p

b) $\det A = \begin{vmatrix} 1 & 1 & 2 \\ 1 & 2 & a \\ -2 & -1 & 3 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 2 \\ 0 & 1 & a-2 \\ 0 & 1 & 7 \end{vmatrix} = 9-a$ 6p

Sist. are soluție unică în $\mathbb{R} \underset{2p}{\overset{\text{Cramer}}{\Longleftrightarrow}} \det A \neq 0 \overset{2p}{\Leftrightarrow} a \in \mathbb{R}\setminus\{9\}$

c) $-x_0+y_0+z_0 = 11(x_0+y_0+z_0) \Leftrightarrow 12x_0+10y_0+10z_0=0 \Leftrightarrow 6x_0+5y_0+5z_0=0$ 2p

Dacă (x_0, y_0, z_0) soluție $\overset{2p}{\Rightarrow} \begin{cases} x_0+y_0+2z_0=0 \\ -2x_0-y_0+3z_0=0 \end{cases} \xrightarrow[(-1)\cdot ec2]{4\cdot ec1} \begin{cases} 4x_0+4y_0+8z_0=0 & 2p \\ 2x_0+y_0-3z_0=0 & 2p \end{cases}$

$6x_0+5y_0+5z_0=0$ c.c.t.d.

2p