Secțiunea 15 – Geometria analitică a dreptei în plan

Exersare of: 10p

Partea I

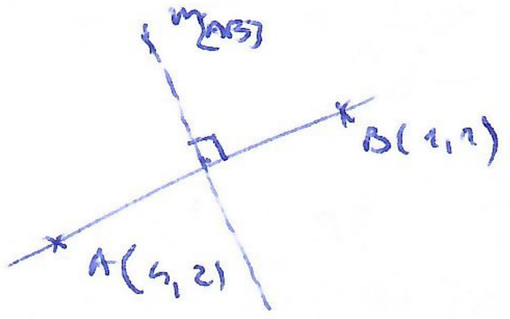
1. $d \parallel Ox \overset{2p}{\Rightarrow} d: y = y_0 \overset{2p}{\Rightarrow} a-1=0 \overset{2p}{\Rightarrow} a=1 \Rightarrow d: y=-1$ 4p

2. $d_1 \perp d_2 \overset{3p}{\Leftrightarrow} m_{d_1} \cdot m_{d_2} = -1 \overset{3p}{\Leftrightarrow} \frac{1}{2}\cdot(m-3) = -1 \Leftrightarrow m = 1$ 4p

3. $ABCD$ paralelogram $\overset{4p}{\Leftrightarrow} \overline{AB} = \overline{DC} \Leftrightarrow -4\vec{i} - 2\vec{j} = -4\vec{i} - 2\vec{j}$ adevărat 6p

Partea a II-a

1. a) $\begin{cases} x_M = \frac{x_A + x_B}{2} = \frac{4+1}{2} = \frac{5}{2}\ 4p \\ y_M = \frac{y_A + y_B}{2} = \frac{2+1}{2} = \frac{3}{2}\ 4p \end{cases} \Rightarrow M\left(\frac{5}{2}, \frac{3}{2}\right)$ 2p



b) $m_{[AB]}: \frac{y-y_M}{x-x_M} = m$ CNN

$m_{[AB]} \perp AB \Leftrightarrow m \cdot m_{AB} = -1 \overset{2p}{} \Rightarrow m_1 = -\frac{1}{m_{AB}} = 3 \overset{2p}{}$

$m_{AB} = \frac{y_B - y_A}{x_B - x_A} = \frac{1-2}{1-4} = -\frac{1}{3} \overset{2p}{}$

$\Rightarrow m_{[AB]}: \frac{y-\frac{5}{2}}{x-\frac{3}{2}} = 3$ CNN 2p

$\Downarrow$

$m_{AB}: 2y-5 = 6x-9 \Leftrightarrow$

$\Leftrightarrow m_{AB}: 6x-2y-4=0$ 2p

2. a) $AB: \frac{y-y_A}{x-x_A} = m_{AB}$ CNN

$m_{NB} = \frac{y_B - y_A}{x_B - x_A} = \frac{-2}{-1} = 2 \overset{2p}{}$

$\Rightarrow AB: y-5 = 2(x-2) \overset{2p}{\Rightarrow} AB: 2x-y+1=0$ 2p

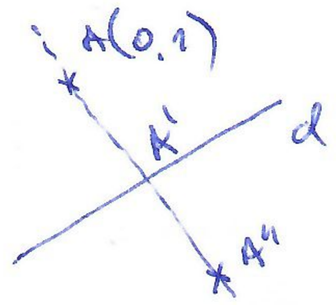
$C \in AB \overset{2p}{\Leftrightarrow} C$ verifică ecuația dreptei $\Leftrightarrow 2m-1+1=0 \Leftrightarrow m=0 \Rightarrow C(0,1)$ 2p

b) $d(C, AB) \overset{4p}{=} \frac{|2x_C - y_C + 1|}{\sqrt{2^2+(-1)^2}} = \frac{|2m-1+1|}{\sqrt{5}} = \frac{2m}{\sqrt{5}} \Rightarrow 2m = \sqrt{5} \Rightarrow m = \frac{\sqrt{5}}{2}$ 6p

3. a) $d: y = x-10 \overset{2p}{\Rightarrow} m_d = 1$

$d' \parallel d \Rightarrow m_{d'} = 1$ 2p $\Big/ \overset{2p}{\Rightarrow} d': \frac{y-y_A}{x-x_A} = 1$ CNN $\overset{2p}{\Rightarrow} d': y-1 = x \Rightarrow d': x-y+1=0$

b)



fie $A' = \sigma_d A$

$A'' = S_{im_d} A$

$AA' \perp d \Rightarrow m_{AA'} = -\frac{1}{m_d} = -1 \overset{2p}{\Rightarrow} AA': \frac{y-y_A}{x-x_A} = -1$ CNN $\Rightarrow$

$\Rightarrow AA': y-1 = -x \Rightarrow AA': x+y=1$ 2p

$\{A'\} = d \cap AA' \overset{2p}{\Rightarrow} A'(x,y)$ verifică și ecuația dreptei d și ecuația dreptei AA'

$\Rightarrow \begin{cases} y = x-10 \\ x+y=1 \end{cases} \Rightarrow \begin{cases} x = \frac{11}{2} \\ y = -\frac{9}{2} \end{cases} \Rightarrow A'\left(\frac{11}{2}, -\frac{9}{2}\right)$ 2p

$A'\left(\frac{11}{2}, -\frac{9}{2}\right)$ mijlocul $AA'' \Rightarrow \begin{cases} \frac{11}{2} = \frac{0+x''}{2} \\ -\frac{9}{2} = \frac{1+y''}{2} \end{cases} \Rightarrow \begin{cases} x'' = 11 \\ y'' = -10 \end{cases} \Rightarrow A''(11,-10)$ 2p

Sectiunea 15 - Geometria analitică a dreptei în plan

Aprofundare 10p

Partea I

1. $d_1: y = ax+2 \Rightarrow m_1 = a$ 3p

$d_2: y = \frac{x}{4}+1 \Rightarrow m_2 = \frac{1}{4}$ 3p

$d_1 \parallel d_2 \Leftrightarrow m_1 = m_2 \Leftrightarrow a = \frac{1}{4}$ 4p

2. M mijloc (BC)

fie $D = sim_M A \overset{2p}{\Rightarrow}$ M mijloc (AD)

$\Rightarrow$ ABDC paralelogram $\overset{2p}{\Rightarrow} \overline{AB} = \overline{CD} \Rightarrow$

$\overset{2p}{\Rightarrow} 3\bar{i} + 3\bar{j} = (x_D+1)\bar{i} + (y_D-3)\bar{j} \overset{2p}{\Rightarrow} \begin{cases} x_D = 2 \\ y_D = 6 \end{cases} \Rightarrow$

$\overset{2p}{\Rightarrow} D(2,6)$

3. $\overline{AB} = \bar{i}$ 3p

$\overline{AC} = 3\bar{i} + (a-1)\bar{j}$ 3p

$\overline{AB} \parallel \overline{AC} \Rightarrow \frac{3}{1} = \frac{a-1}{0}$ CNN 3p $\Rightarrow \boxed{a = 1}$ 1p

Partea a II-a

1.a) $P \in Ox \rightarrow P(x,0)$ 4p

$PM = PN \Rightarrow \sqrt{(x-2)^2+(-2)^2} \overset{3p}{=} \sqrt{(x-4)^2+(-2)^2} \Rightarrow (x-2)^2 = (x-4)^2 \Rightarrow x = 3 \Rightarrow P(3,0)$ 3p

b) $Q \in b_1 \Rightarrow Q(x,x)$ 4p

$QM = QN \Rightarrow (x-2)^2 + (-2)^2 \overset{3p}{=} (x-4)^2 + (x-2)^2 \Rightarrow x = 3 \Rightarrow Q(3,3)$ 3p

2 a) $d: y = 3x - 2015 \Rightarrow m_d = 3$ 3p

$d' \parallel d \Rightarrow m_{d'} = m_d = 3$ 3p $\Rightarrow d': \frac{y-y_A}{x-x_A} \overset{2p}{=} 3$ CNN $\Rightarrow d': y = 3(x-1) \Rightarrow d': 3x - y = 3$ 2p

b) $d_2: y = 3x - 2 \overset{2p}{\Rightarrow} m_{d_2} = 3 \Rightarrow m_{d_2} = m_d \Rightarrow d \parallel d_2$ 2p

Alegi $A(0,-2) \in d_2$ 2p

$d(d_2, d) = d(A, d) \overset{2p}{=} \frac{|y_A - 3x_A + 2015|}{\sqrt{1^2+3^2}} = \frac{2017}{\sqrt{10}} = \frac{2017\sqrt{10}}{10}$ 2p

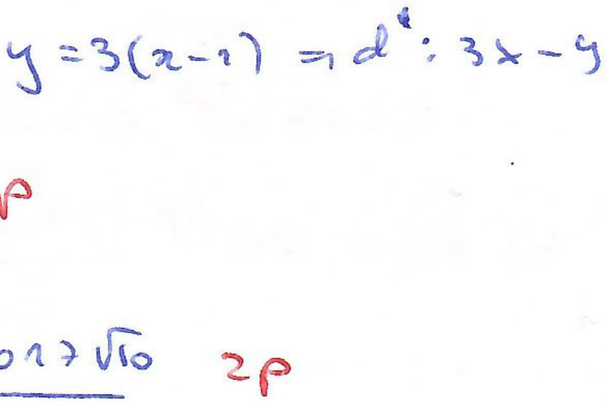
3 a) $M = sim_B A \overset{2p}{\Rightarrow} B$ mijloc $(AM) \Rightarrow \begin{cases} 4 = \frac{2+x}{2} \\ 1 = \frac{2+y}{2} \end{cases} \Rightarrow \begin{cases} x = 6 \\ y = 0 \end{cases} \Rightarrow M(6,0)$ 4p

$\Rightarrow AM = \sqrt{(6-2)^2+(0-2)^2} = 2\sqrt{5}$ 4p

b) fie O ccc $\triangle ABC \Rightarrow \{O\} = m_{[AC]} \cap m_{[AB]}$ 2p

$m_{AC} = -\frac{1}{3} \Rightarrow m_{m_{[AC]}} = 3$

$m_{AB} = -2 \Rightarrow m_{m_{[AB]}} = \frac{1}{2}$



$m_{[AC]}: \frac{y-5}{x-1} \overset{2p}{=} 3$ CNN

$m_{[AB]}: \frac{y-\frac{3}{2}}{x-3} \overset{2p}{=} \frac{1}{2}$ CNN $\Rightarrow$ O verifică $\begin{cases} y = 3x+2 \\ x - 2y = 0 \end{cases}$ 2p $\Rightarrow \begin{cases} x = -\frac{4}{5} \\ y = -\frac{2}{5} \end{cases} \Rightarrow O(-\frac{4}{5}, -\frac{2}{5})$ 2p