Secțiunea 13 – Mulțimea numerelor complexe

Exersare of: 10p

Partea I

1. $1+i+(i-1)(1+i)-(i-1)=1+i+i^2-1-i+1=i^2+1=-1+1=0$ 10p

2. $(5+4i)^2+(5-4i)^2=(5+4i)^2+\overline{(5-4i)^2}=2\,\mathrm{Re}\left((5-4i)^2\right)\in\mathbb{R}$ 10p

3. $z=1-i\Rightarrow z^2=(1-i)^2=1^2-2i+i^2=-2i\Rightarrow z^2+2i=(-2i)+2i=0$ 10p

Partea a II-a

1. a) $z^2=(1-2i)^2=1-4i+4i^2=-3-4i$ 5p

$z^2-2z+5=-3-4i-2(1-2i)+5=-3-2+5-4i+4i=0$ 5p

b) $x^2-2x+5=0$ are coeficienți reali 3p

$x_1=1-2i$ este o rădăcină imaginară 3p

$\Rightarrow x_2=1+2i \overset{2p}{\Rightarrow} S=\{1-2i,\ 1+2i\}$ 2p

2. a) $1-i\overset{4p}{=}\sqrt{2}\left(\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}i\right)\overset{4p}{=}\sqrt{2}\left(\cos\frac{7\pi}{4}+i\sin\frac{7\pi}{4}\right)$, $\frac{7\pi}{4}\in[0,2\pi)$ 2p

b) $z^2=(1-i)^2=-2i\overset{4p}{\Rightarrow} z^{2020}\overset{2p}{=}(z^2)^{1010}=(-2i)^{1010}=2^{1010}\cdot i^{1010}\overset{2p}{=}2^{1010}\cdot i^2=-2^{1010}$ 2p

3. a) $\varepsilon^2+\varepsilon+1=0\overset{4p}{\Rightarrow}(\varepsilon-1)(\varepsilon^2+\varepsilon+1)=0\overset{4p}{\Rightarrow}\varepsilon^3-1=0\Rightarrow\varepsilon^3=1$ 2p

b) $\varepsilon^{2019}\overset{2p}{=}\varepsilon^{3\cdot 673}=1^{673}\overset{2p}{=}1$

$\varepsilon^{2020}=\varepsilon^{2019}\cdot\varepsilon\overset{2p}{=}\varepsilon$

$\Rightarrow \varepsilon^{2020}+\varepsilon^{2019}\overset{2p}{=}\varepsilon+1=\frac{1+i\sqrt{3}}{2}$ 2p

Secțiunea 13 — Numere complexe

Aprofundare of: 10p

Partea I

1. $3z_1 + 2z_2 = 3(5+2i) + 2(3-3i) = 15+6i + 6-6i = 21$ 10p

2. $2\bar{z} - z = 1-3i \overset{\overline{()}\ 3p}{\Longrightarrow} 2z - \bar{z} = 1+3i \Rightarrow 4z - 2\bar{z} = 2+6i$ 3p

Dar $-z + 2\bar{z} = 1-3i$

$3z = 3+3i \Rightarrow z = 1+i$ 4p

3. $z + \bar{z} + z\cdot\bar{z} \overset{5p}{=} 2\,\mathrm{Re}\,z + |z|^2 = 2\cdot 2 + \sqrt{2^2+1^2}^2 = 4 + \sqrt{5}^2 = 9$ 5p

Partea a II-a

1. a) $|w| = \left|\frac{z_1^{2020}}{z_2^{2019}}\right| \overset{5p}{=} \frac{|z_1|^{2020}}{|z_2|^{2019}} = \frac{\sqrt{3^2+4^2}^{\,2020}}{\sqrt{4^2+(-3)^2}^{\,2019}} = 5$ 5p

b) Notăm $\delta = a+bi$ 2p

$\delta^2 = z_1 \Leftrightarrow a^2 - b^2 + 2abi = 3+4i \Leftrightarrow \begin{cases} a^2-b^2 = 3 \\ 2ab = 4 \end{cases}$ 2p $\overset{a\neq 0}{\Longrightarrow} b = \frac{2}{a}$ $\Bigg| \Rightarrow$

$\Rightarrow a^2 - \frac{4}{a^2} = 3 \Rightarrow a^4 - 3a^2 - 4 = 0 \Rightarrow a^2 \in \{4, -1\}$

Dar $a^2 \geq 0$ $\quad a^2 = 4 \Rightarrow a = \pm 2$

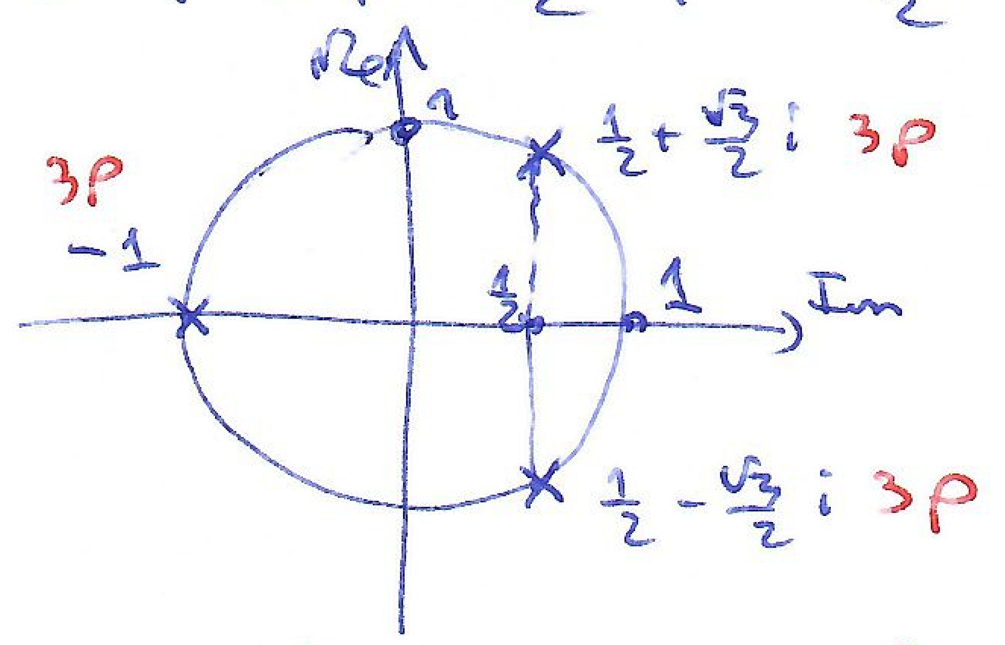
Pentru $a = 2 \Rightarrow b = 1 \Rightarrow \delta = 2+i$ 2p

Pentru $a = -2 \Rightarrow b = -1 \Rightarrow \delta = -2-i$ 2p $\Rightarrow S = \{\pm(2+i)\}$ 2p

2. a) $z^3 + 1 = 0 \overset{2p}{\Leftrightarrow} (z+1)(z^2 - z + 1) = 0 \Rightarrow z = -1$ 2p sau $z^2 - z + 1 = 0 < \frac{1 \pm \sqrt{3}i}{2}$ 4p

$\Rightarrow S = \left\{-1, \frac{1-\sqrt{3}i}{2}, \frac{1+\sqrt{3}i}{2}\right\}$ 2p

b)




+1p respectare unități măsură

3. a) $z = \sqrt{3} + i \overset{4p}{=} 2\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right) \overset{4p}{=} 2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$, $\frac{\pi}{6} \in [0, 2\pi)$ 2p

b) $\sum_{k=0}^{2019} (z - \sqrt{3})^k \overset{2p}{=} \sum_{k=0}^{2019} i^k \overset{4p}{=} (i^0 + i^1 + i^2 + i^3) + (i^4 + i^5 + i^6 + i^7) + \ldots$

$+ (i^{2016} + i^{2017} + i^{2018} + i^{2019}) = 0$ 4p