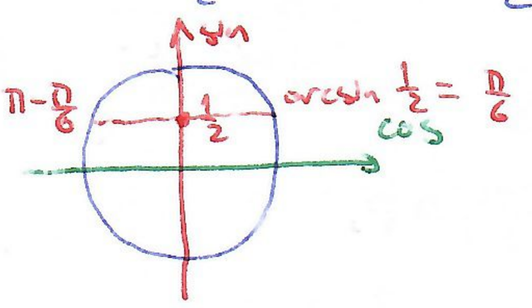
Sectiunea 12 - Inversele functiilor trigonometrice

Exercare of 10p

Partea I

1. $\arcsin\frac{1}{2} - \arccos\frac{\sqrt{3}}{2} = \frac{\pi}{6} - \frac{\pi}{6} = 0$ 3p 3p 4p

2. 

$\sin x = \frac{1}{2}$, $x \in [0, 2\pi)$ 3p $\Rightarrow x \in \{\arcsin\frac{1}{2} = \frac{\pi}{6}, \pi - \frac{\pi}{6} = \frac{5\pi}{6}\} \Rightarrow$ 3p

$\Rightarrow S = \{\frac{\pi}{6}, \frac{5\pi}{6}\}$ 1p

3. $\operatorname{tg} x + 1 = 0 \Leftrightarrow \operatorname{tg} x = -1$ 2p $\Leftrightarrow x \in \{\operatorname{arctg}(-1) + k\pi\}$ 2p

$x \in \{\pi, 3\pi)$ 2p $\Rightarrow x \in \{-\frac{\pi}{4} + 2\pi, -\frac{\pi}{4} + 3\pi\}$ 2p

$\Rightarrow S = \{\frac{3\pi}{4}, \frac{11\pi}{4}\}$ 2p

Partea a II-a

1. a) CE: $-1 \le x^2 - 1 \le 1 \Leftrightarrow x^2 \in \{0, 2\} \Leftrightarrow x \in [-\sqrt{2}, \sqrt{2}]$ 4p

$-1 \le \sqrt{x} \le 1 \Leftrightarrow x \in [-1, 1]$ 4p $\Rightarrow x \in \{-1, 1\} \Rightarrow$ 2p

b) $f(0) = \arcsin(-1) + \arccos 0 = -\frac{\pi}{2} + \frac{\pi}{2} = 0$ 4p

$f(1) = \arcsin 0 + \arccos 1 = 0 + 0 = 0$ 4p $\Rightarrow f(0) + f(\pi) = 0$ 2p

2. a) $f(1) = \sin 1 - \cos 0 = \sin 1 - 1$ 10p

b) $f(x) = 0 \Leftrightarrow \sin x = \cos(x-1) \Leftrightarrow \sin\left(\frac{\pi}{2} - x\right) = \sin(x-1)$ 2p $\Leftrightarrow$

$\Leftrightarrow \frac{\pi}{2} - x = (x-1)\cdot(-1)^n + n\pi$

(I) n impar $\Rightarrow \frac{\pi}{2} - x = -x + 1 + n\pi$ fals $\forall x \in \mathbb{R}$ 3p

(II) n par $\Rightarrow \frac{\pi}{2} - x = x - 1 + n\pi \Rightarrow x = \frac{1}{2}\left(\frac{\pi}{2} + 1 - n\pi\right)$, $n \in \mathbb{Z}$ $\Rightarrow$ 3p

$\Rightarrow S = \{\frac{1}{2}(\frac{\pi}{2} + 1 + n\pi \mid n \in \mathbb{Z}\}$ 2p

3. a) $f(-1) = \cos(-1) + \sin(-1) = \cos 1 - \sin 1$ 10p

b) $f(x) = 1 \Leftrightarrow \sin x + \cos x = 1 \Leftrightarrow \sqrt{2}\left(\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x\right) = 1$ 3p $\Leftrightarrow$

$\Leftrightarrow \sin\left(x + \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} \Leftrightarrow x + \frac{\pi}{4} = (-1)^n \arcsin\frac{1}{\sqrt{2}} + n\pi$ 2p

(I) n par $\Rightarrow x = n\pi$, $n \in \mathbb{Z}$ 2p

(II) n impar $\Rightarrow x + \frac{\pi}{4} = -\frac{\pi}{4} + n\pi$, $n \in \mathbb{Z} \Rightarrow x = -\frac{\pi}{2} + n\pi$, $n \in \mathbb{Z}$ 2p $\Rightarrow$

$\Rightarrow S = \{n\pi, -\frac{\pi}{2} + n\pi \mid n \in \mathbb{Z}\} = \{0, -\frac{\pi}{2}\} + \pi\mathbb{Z}$ 1p

notatie

Sectiunea 12 — Inversele functiilor trigonometrice

Aprofundare Of 10p

Partea I

1. $\sin(\arcsin\frac{2}{3}) - \sin(\arccos\frac{1}{3}) = \frac{2}{3} - \sqrt{1-(\frac{1}{3})^2}$ 3p $= \frac{2}{3} - \frac{2\sqrt{2}}{3} = \frac{2(1-\sqrt{2})}{3}$ 4p

2. $\cos 2x = -\frac{1}{2} \Leftrightarrow 2x = \pm\arccos(-\frac{1}{2}) + 2k\pi$ 2p $\Leftrightarrow 2x = \pm\frac{2\pi}{3} + 2k\pi \Leftrightarrow x = \pm\frac{\pi}{3} + k\pi$ 2p

Dar $x \in [0, 2\pi)$ 2p $\Rightarrow x \in \{0+\frac{\pi}{3}, 2\pi - \frac{\pi}{3}\} = \{\frac{\pi}{3}, \frac{5\pi}{3}\}$ 2p

3. $\sin^2 x = 1 - \cos^2 x$ 2p $\Rightarrow$ ecuatia devine $4\cos x + 4(1-\cos^2 x) + 1 = 5$

Notez $\cos x = t$ si obtin $4t + 4 - 4t^2 = 5 \Leftrightarrow 4t^2 - 4t + 1 = 0$ 2p

$\Leftrightarrow (2t-1)^2 = 0 \Leftrightarrow t = \frac{1}{2}$ 2p $\Leftrightarrow \cos x = \frac{1}{2} \Leftrightarrow x \in \{\pm\arccos\frac{1}{2} + 2k\pi \mid k \in \mathbb{Z}\}$ 2p.

$S = \{\pm\frac{\pi}{3} + 2k\pi \mid k \in \mathbb{Z}\}$ 2p

Partea a II-a

1. a) CE: $x \in [-1, 1] = D$ 1p

b) $\arcsin x + \arccos x = \frac{\pi}{2} \Leftrightarrow \underbrace{\arcsin x}_{\in[-\frac{\pi}{2}, \frac{\pi}{2}]} = \underbrace{\frac{\pi}{2} - \arccos x}_{\in[-\frac{\pi}{2}, \frac{\pi}{2}]}$ 3p $(\Leftrightarrow)$

$\Leftrightarrow \sin(\arcsin x) = \sin(\frac{\pi}{2} - \arccos x)$ 3p $\Rightarrow$

$\Rightarrow x = \cos(\arccos x)$ 4p adevarat $\forall x \in \mathbb{R}$

2. a) $f(-\frac{\pi}{2}) = \sin(-\frac{\pi}{2}) - \frac{\sqrt{3}}{2}\cos(-\frac{\pi}{2}) = -\sin\frac{\pi}{2} - \frac{\sqrt{3}}{2}\cos\frac{\pi}{2} = -1$ 1p

b) $f(x) = 1 \Leftrightarrow \sin x - \sqrt{3}\cos x = 1 \Leftrightarrow 2(\frac{1}{2}\sin x - \frac{\sqrt{3}}{2}\cos x) = 1$ 2p $\Leftrightarrow$

$\Leftrightarrow \cos\frac{\pi}{3}\sin x - \sin\frac{\pi}{3}\cos x = \frac{1}{2}$ 2p $\Leftrightarrow \sin(x - \frac{\pi}{3}) = \frac{1}{2}$ 2p $\Leftrightarrow$

$\Leftrightarrow x - \frac{\pi}{3} = (-1)^n \arcsin\frac{1}{2} + n\pi, n \in \mathbb{Z}$ 1p $\Leftrightarrow$

$\Leftrightarrow x = \frac{\pi}{3} + (-1)^n \cdot \frac{\pi}{6} + n\pi, n \in \mathbb{Z}$ 1p $\Leftrightarrow$

$\Leftrightarrow x \in \{\frac{\pi}{2} + 2k\pi, \frac{5\pi}{6} + 2k\pi \mid k \in \mathbb{Z}\}$ 2p

3. a) $\sqrt{3} = \sqrt[6]{27}$ 1p
$\sqrt[3]{5} = \sqrt[6]{81}$ 1p $\Rightarrow$ $0 < \frac{1}{\sqrt{3}} < \sqrt{3} < \sqrt[3]{5} < \sqrt[3]{27} = 3 < \pi$ 3p

$f = \cos$ $\searrow$ pe $[0, \pi]$ 2p $\Rightarrow$

$\Rightarrow \cos\frac{1}{\sqrt{3}} < \cos\sqrt{3} < \cos\sqrt[3]{5}$ 3p $\Rightarrow f(\frac{1}{\sqrt{3}}) < f(\sqrt{3}) < f(\sqrt[3]{5})$

b) fie $h(x) = x + \cos x$, $h: [0, \pi] \to \mathbb{R}$

$h': [0, \pi] \to \mathbb{R}$, $h'(x) = 1 - \sin x \geq 0 \quad \forall x \in (0, \pi) \Rightarrow h \nearrow$ pe $[0, \pi]$ 2p $\Rightarrow$

$\Rightarrow h$ injectiva 2p

Asadar

$x + \cos x = \underline{1}$ are solutie unica $x \in (0, \pi)$ 2p

Observam $x = 0$ solutie 2p $\Rightarrow x = 0$ UNICA SOLUTIE 2p