Secțiunea 11 – Funcția exponențială. Funcția logaritm

Exersare of: 10 p

Partea I

1. $25^x = (5^2)^x = 5^{2x}$ 2p

$25^x = 5^{x^2} \Leftrightarrow 5^{2x} = 5^{x^2}$ 2p $\Leftrightarrow 2x = x^2$ 2p $\Leftrightarrow x(x-2) = 0$ 2p $\Leftrightarrow x \in \{0, 2\}$ 2p

2. CE: $x^2 - 5x + 7 > 0$ 2p (A) $(\forall) x \in \mathbb{R}$

$\log_2(x^2-5x+7) = \log_2 3 \Leftrightarrow x^2 - 5x + 7 = 3$ 2p $\Leftrightarrow x^2 - 5x + 4 = 0$ 2p $\Leftrightarrow x \in \{1, 4\}$ 4p

3. $2^{2x+1} = \frac{1}{8} \Leftrightarrow 2^{2x+1} = 2^{-3}$ 4p $\Leftrightarrow 2x+1 = -3$ 4p $\Leftrightarrow x = -2$ 2p

Partea a II-a

1. CE: $x > 0$ 2p

a) $x \nearrow$ pe $(0, \infty)$ 2p

$\log_2 x \nearrow$ pe $(0,\infty)$ pt că $2 > 1$ 2p

$3^x \nearrow$ pe $(0, \infty)$ pt că $3 > 1$ 2p

$\Rightarrow f(x) \nearrow$ pe $(0, \infty)$ 2p

b) $\underbrace{x + \log_2 x + 3^x}_{\nearrow \text{ deci injectivă } 3p} = 4$

Obs $x = 1$ soluție 3p

$\Rightarrow x = 1$ unica soluție 4p

2. a) CE: $x + 3 > 0 \Leftrightarrow x > -3$ 3p

$x - 3 > 0 \Leftrightarrow x > 3$ 3p

$\Rightarrow D = (3, \infty)$ 4p

b) $f(x) = 2 \Leftrightarrow \log_4(x+3) + \log_4(x-3) = 2 \Leftrightarrow \log_4\left((x+3)(x-3)\right) = \log_4 4^2$ 2p $\Leftrightarrow$

$\Leftrightarrow (x+3)(x-3) = 4^2$ 2p $\Leftrightarrow x^2 - 6x + 9 = 16 \Leftrightarrow x^2 - 6x - 7 = 0 \Leftrightarrow x \in \{7, -1\}$ 2p

Dar $x \in D = (3, \infty)$ 2p $\Rightarrow \boxed{x = 7}$ 2p

3. a) a, b, c ÷÷ $\Leftrightarrow b^2 = ac$ 2p $\Leftrightarrow (\log_4 x)^2 = \log_2 x \cdot \log_{16} x$ Adevărat deoarece 2p

$(\log_4 x)^2 = (\log_{2^2} x)^2 = \left(\frac{1}{2}\log_2 x\right)^2 = \frac{1}{4}\log_2^2 x$ 3p

$\log_2 x \cdot \log_{16} x = \log_2 x \cdot \log_{2^4} x = \frac{1}{4}\log_2^2 x$ 3p

b) $a + b + c = \log_2 x + \frac{1}{2}\log_2 x + \frac{1}{4}\log_2 x = \left(1 + \frac{1}{2} + \frac{1}{4}\right)\log_2 x = \frac{7}{4}\log_2 x$ 4p

$a + b + c = 7 \Leftrightarrow \frac{7}{4}\log_2 x = 7 \Leftrightarrow \log_2 x = 4$ 2p $\Leftrightarrow x = 2^4$ 2p $\Leftrightarrow \boxed{x = 16}$ 2p

Secţiunea 11 — Funcţii exponenţiale. Funcţii logaritm

Aprofundare 10p

Partea I:

1. $3^{x^2-3x}=3^{x-4} \Leftrightarrow x^2-3x=x-4 \overset{3p}{\Leftrightarrow} x^2-4x+4=0 \overset{3p}{\Leftrightarrow} x=2$ 4p

2. $9^x-6^{x+1}+5\cdot 4^x=0 \overset{1:4^x}{\Leftrightarrow} \frac{9^x}{4^x}-6\cdot\frac{6^x}{4^x}+5=0 \overset{2p}{\Leftrightarrow} \left(\frac{3}{2}\right)^{2x}-6\left(\frac{3}{2}\right)^x+5=0$ 2p

Notăm $\left(\frac{3}{2}\right)^x=t>0$ 2p.

$t^2-6t+5=0 \Rightarrow t\in\{1,5\}$ 2p $\Rightarrow x\in\{0, \log_{\frac{3}{2}}5\}$ 2p

3. CE: $x>0$ 2p

$\log_2 x=1 \Rightarrow x=2$ 3p sau $\log_2 x=2 \Rightarrow x=4$ 3p $\Rightarrow x\in\{2,4\}$ 2p

Partea a II-a:

1. a) CE: $x^2-4>0 \Rightarrow x\in(-\infty,-2)\cup(2,\infty)$ 3p

$5x-8>0 \Rightarrow x>\frac{8}{5}$ 3p

$\Rightarrow x\in(2,\infty)$ 4p

b) $f(x)=0 \Leftrightarrow \log_5(x^2-4)=\log_5(5x-8) \overset{2p}{\Leftrightarrow} x^2-4=5x-8$ 2p $\Leftrightarrow$

$\Leftrightarrow x^2-5x+4=0 \Rightarrow x\in\{1,4\}$ 2p

Dar $x>2$ 2p $\Rightarrow x=4$ unica soluţie 2p

2. a) $\ln^2 x^2 \overset{3p}{=} (\ln x^2)^2 \overset{3p}{=} (2\ln x)^2 \overset{4p}{=} 4\ln^2 x \quad \forall x>0$

b) $\ln^2 x^2+4\ln x+1=0 \Leftrightarrow 4\ln^2 x+4\ln x+1=0 \overset{2p}{\Leftrightarrow} (2\ln x+1)^2=0$ 2p

$\Leftrightarrow 2\ln x+1=0$ 2p $\Leftrightarrow \ln x=-\frac{1}{2}$ 2p $\Leftrightarrow x=e^{-\frac{1}{2}} \Leftrightarrow x=\sqrt{e}$ 2p

3. a) $(2+\sqrt{3})(2-\sqrt{3})=2^2-\sqrt{3}^2=4-3=1$ 10p

b) $(2+\sqrt{3})^x+(2-\sqrt{3})^x=14 \Leftrightarrow (2+\sqrt{3})^x+\left(\frac{1}{2+\sqrt{3}}\right)^x=14$ 2p $\Rightarrow$

Notăm $(2+\sqrt{3})^x=t>0 \Rightarrow$ ecuaţia devine $t+\frac{1}{t}=14$ 2p $\Rightarrow$

$\Rightarrow t^2-14t+1=0 \Leftrightarrow t=7\pm4\sqrt{3}$ 2p

Cum $7+4\sqrt{3}=(2+\sqrt{3})^2$ şi $7-4\sqrt{3}=(2-\sqrt{3})^2=(2+\sqrt{3})^{-2}$

din $(2+\sqrt{3})^x=7+4\sqrt{3} \Rightarrow (2+\sqrt{3})^x=(2+\sqrt{3})^2 \Rightarrow x=2$ 2p

şi $(2+\sqrt{3})^x=7-4\sqrt{3} \Rightarrow (2+\sqrt{3})^x=(2+\sqrt{3})^{-2} \Rightarrow x=-2$ 2p

Aşadar $S=\{-2,2\}$