Secțiunea 10 – Funcția putere. Funcția radical

Exersare of: 10 p

Partea I

1. $\sqrt[3]{-64} + 27^{\frac{2}{3}} + \{\sqrt[3]{4}\} = \sqrt[3]{(-2)^6} + (3^3)^{\frac{2}{3}} + \overset{2p}{1} = \sqrt[3]{2^6} + 3^{3\cdot\frac{2}{3}} \overset{2p}{+} 1 = 2^2 + 3^2 + \overset{2p}{1} = 14$ 2p

$1^3 < 4 < 2^3 \Rightarrow 1 < \sqrt[3]{4} < 2 \Rightarrow \{\sqrt[3]{4}\} = 1$ 2p

2. CE: $x+3 \geq 0 \Rightarrow x \geq -3$

CC: $x-3 \geq 0 \Rightarrow x \geq 3$ $\Rightarrow x \in [3,\infty)$ 2p $\Rightarrow S = \{6\}$ 2p

$\sqrt{x+3} = x-3 \Rightarrow x+3 = x^2-6x+9 \overset{2p}{\Rightarrow} x^2-7x+6=0 \Rightarrow$ $1 < 3$ 2p; $6 \geq 3$ 2p

3. $\sqrt{x^2-x-2} = \sqrt{x-2}$

CE: $x^2-x-2 \geq 0 \Rightarrow x \in (-\infty,-1] \cup [2,\infty)$

$x-2 \geq 0 \Rightarrow x \geq 2$ $\Rightarrow x \in [2,\infty)$ 2p

$\sqrt{x^2-x-2} = \sqrt{x-2} \Rightarrow x^2-x-2 = x-2 \overset{2p}{\Leftrightarrow} x^2-2x=0 \overset{2p}{\Leftrightarrow} x \in \{0,2\}$ 2p $\Rightarrow$

$\Rightarrow x = 2$ unica soluție 2p

Partea a II-a

1. a) $(1+\sqrt{2})^3 = 1^3 + 3\cdot 1^2\cdot\sqrt{2} + 3\cdot 1\cdot\sqrt{2}^2 + \sqrt{2}^3 = 7+5\sqrt{2}$ 4p $\Rightarrow$

$\Rightarrow \sqrt[3]{7+5\sqrt{2}} = \sqrt[3]{(1+\sqrt{2})^3} \overset{2p}{=} |1+\sqrt{2}| = \sqrt{2}+1$ 4p

b) $7+5\sqrt{2} = (\sqrt{2}+1)^3$

$(\sqrt{2}+1)(\sqrt{2}-1) = 1 \Rightarrow \sqrt{2}+1 = (\sqrt{2}-1)^{-1}$ 4p $\Rightarrow (7+5\sqrt{2}) = (\sqrt{2}-1)^{-3}$ 4p $\Rightarrow \boxed{x=-3}$ 2p

2. a) $E(4) = [\sqrt{4}] = [2] = 2$ 3p

$E(9) = [\sqrt{9}] = [3] = 3$ 3p

$E(5) = [\sqrt{5}] = 2$ deoarece

cum $4 < 5 < 9 \Rightarrow 2 < \sqrt{5} < 3 \Rightarrow [\sqrt{5}] = 2$ 4p

b) $\sum_{k=1}^{10} E(k) = \underset{1}{[\sqrt{1}]} + [\sqrt{2}] + [\sqrt{3}] + \underset{2}{[\sqrt{4}]} + [\sqrt{5}] + [\sqrt{6}] + [\sqrt{7}] + [\sqrt{8}] + \underset{3}{[\sqrt{9}]} + [\sqrt{10}]$ 4p

$= 1\cdot 3 + 2\cdot 5 + 3\cdot 2 = 19$ 6p

3. a) $x-1 \nearrow$ pe $\mathbb{R}$

$\sqrt[3]{} \nearrow$ pe $\mathbb{R}$ $\Rightarrow f(x) = \sqrt[3]{x-1} \nearrow$ pe $\mathbb{R}$ 4p $\Rightarrow f(\frac{\pi}{2}) < f(\sqrt[3]{9}) < f(\sqrt{5})$ 2p

$\sqrt{5} \boxed{?} \sqrt[3]{9} \overset{()^6}{\Leftrightarrow} 5^3 \boxed{?} 9^2 \Leftrightarrow 125 > 81$ 2p $\Rightarrow \frac{\pi}{2} < \sqrt[3]{9} < \sqrt{5}$

$\frac{\pi}{2} < \frac{4}{2} = 2 = \sqrt[3]{8} < 9$ 2p

b) $f(x) = 1-x \Leftrightarrow \sqrt[3]{x-1} = 1-x \Leftrightarrow (x-1) = (1-x)^3 \overset{2p}{\Leftrightarrow} (x-1) + (x-1)^3 = 0$ 2p

$\Leftrightarrow (x-1)(\underbrace{1+(x-1)^2}_{\geq 1\ \forall x}) = 0 \overset{2p}{\Leftrightarrow} x = 1$ unica soluție 4p

Sectiunea 10 – Funcția putere. Funcția radical

Aprofundare of 10p

Partea I

1. $(x+2)^3=(2-x)^3 \Leftrightarrow x+2=2-x \Leftrightarrow x=0$ 10p

2. $\sqrt{x^2-1}=x+1$

CE: $x^2-1\geq 0 \Leftrightarrow x\in(-\infty,-1]\cup[1,\infty)$

CC: $x+1\geq 0 \Leftrightarrow x\geq -1$

$\Rightarrow x\in[1,\infty)\cup\{-1\}$ 2p

$\sqrt{x^2-1}=x+1 \Rightarrow x^2-1=x^2+2x+1 \overset{2p}{\Leftrightarrow} x=-1^{2p}$ verifică CE $\Rightarrow S=\{-1\}$ 4p

3. $\sqrt{x+1}=1-\sqrt{x}$

CE: $x\geq -1$

CC: $1-\sqrt{x}\geq 0 \Leftrightarrow \sqrt{x}\leq 1 \Leftrightarrow x\in[0,1]$

$\Rightarrow x\in[0,1]$ 2p

$\sqrt{x+1}=1-\sqrt{x} \Rightarrow x+1=1-2\sqrt{x}+x \overset{2p}{\Leftrightarrow} 2\sqrt{x}=0 \Rightarrow x=0^{2p}$

$\Rightarrow x=0$ unica soluție 4p

Partea a II-a

1. a) $x-2\sqrt{x-1}=x-1-2\sqrt{x-1}+1=(\sqrt{x-1}-1)^2 \quad \forall x\geq 1 \overset{4p}{\Rightarrow}$

$\Rightarrow E(x)=\sqrt{x-2\sqrt{x-1}}=\sqrt{(\sqrt{x-1}-1)^2} \overset{4p}{=} |\sqrt{x-1}-1|$ 2p $\forall x\geq 1$

b) $E(x)=1 \Rightarrow |\sqrt{x-1}-1|=1 \Rightarrow \sqrt{x-1}-1=\pm 1 \overset{2p}{\Rightarrow} \sqrt{x-1}\in\{0,2\}$ 2p

$\sqrt{x-1}=0 \Rightarrow x=1$ 3p verificare: $E(1)=\sqrt{1-2\sqrt{1-1}}=\sqrt{1}=1$

$\sqrt{x-1}=2 \Rightarrow x=5$ 3p verificare: $E(5)=\sqrt{5-2\sqrt{5-1}}=\sqrt{1}=1$

2. a) $E(8)=[\sqrt[3]{8}]=[2]=2$ 3p

$E(27)=[\sqrt[3]{27}]=[3]=3$ 3p

$8<9<27 \overset{2p}{\Rightarrow} 2<\sqrt[3]{9}<3 \Rightarrow [\sqrt[3]{9}]=2$ 2p

b) $\sum_{k=1}^{100} E(k)=\sum_{k=1}^{100}[\sqrt[3]{k}]=[\sqrt[3]{1}]+[\sqrt[3]{2}]+ \dots +[\sqrt[3]{7}]$

$+[\sqrt[3]{8}]+[\sqrt[3]{9}]+ \dots +[\sqrt[3]{26}]$

$+[\sqrt[3]{27}]+[\sqrt[3]{28}]+ \dots +[\sqrt[3]{63}]$

$+[\sqrt[3]{64}]+ \dots +[\sqrt[3]{100}] \overset{6p}{=}$

$=1\cdot 7+2\cdot 19+3\cdot 37+4\cdot 37 \overset{3p}{=} 304$ 1p

3. a) $E(x)^3-F(x)^2=(\sqrt[3]{x-1})^3-\sqrt{x}^2=x-1-x=-1$ independentă de x 10p

b) $\sqrt[3]{x-1}+\sqrt{x}+1=0$

Notez $\sqrt[3]{x-1}=A$

$\sqrt{x}=B$, $x\geq 0$

$\Rightarrow \begin{cases} A+B=-1 \\ A^3+B^2=-1 \end{cases}$ 2p $\Rightarrow A=-B-1$ 2p $\Rightarrow$

$\Rightarrow -(B+1)^3+B^2=-1 \Rightarrow -[B^3+3B^2+3B+1]+B^2=-1 \Leftrightarrow$

$\Leftrightarrow B^3+2B^2+3B=0 \overset{2p}{\Leftrightarrow} B(\underbrace{B^2+2B+3}_{>0\ \forall B}) = 0$ 2p $\Rightarrow B=0 \Rightarrow \sqrt{x}=0 \Rightarrow x=0$ 2p

verificare: $\sqrt[3]{0-1}+\sqrt{0}+1=0$ Adev.