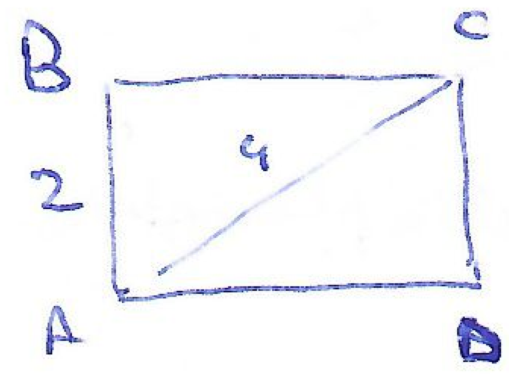
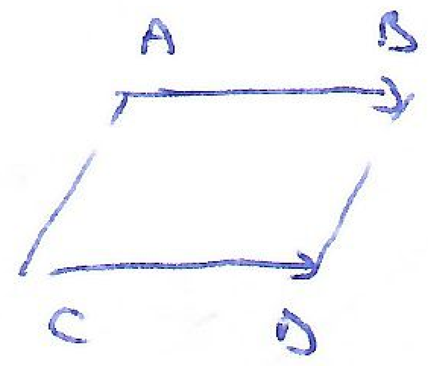
Secțiunea 7 - Vectori liberi în plan

Exersare of 10p

Partea I

1. 

$|\overline{AB}+\overline{BC}| \overset{5p}{=} |\overline{AC}| = 4$ 5p



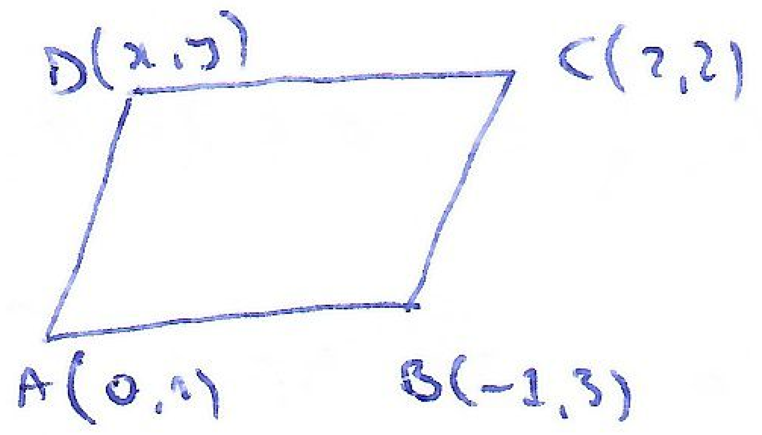
2. $\overline{AB} = \overline{CD} \overset{3p}{\Leftrightarrow}$ ABDC paralelogram $\overset{3p}{\Rightarrow}$

$\Rightarrow \overline{AC} + \overline{DB} = \overline{AC} + (-\overline{BD}) = \overline{AC} - \overline{AC} = \overline{0}$ 4p

3. $\overline{u} = \alpha\overline{a} + \beta\overline{b} \Leftrightarrow 5\overline{i} - \overline{j} = \alpha(\overline{i}+\overline{j}) + \beta(\overline{i}-\overline{j})^{3p} \Leftrightarrow 5\overline{i} - \overline{j} = (\alpha+\beta)\overline{i} + (\alpha-\beta)\overline{j}$ 3p

$\Leftrightarrow \begin{cases} \alpha+\beta = 5 \\ \alpha-\beta = -1 \end{cases} \Leftrightarrow \begin{cases} \alpha = 2 \\ \beta = 3 \end{cases}$ 4p

Partea a II-a

1. 

a) ABCD paralelogram $\overset{2p}{\Leftrightarrow} \overline{AD} = \overline{BC}$

$\overline{AD} \overset{D-A}{=} (x-0)\overline{i} + (y-1)\overline{j}$ 3p

$\overline{BC} \overset{C-B}{=} (2-(-1))\overline{i} + (2-3)\overline{j} = 3\overline{i} - \overline{j}$ 3p

$\Rightarrow \begin{cases} x = 3 \\ y = 0 \end{cases} \Rightarrow D(3,0)$ 2p

b) $\overline{AB} \overset{B-A}{=} -\overline{i} + 2\overline{j} \Rightarrow AB = \|\overline{AB}\| = \sqrt{5}$ 2p

$\overline{AD} \overset{D-A}{=} 3\overline{i} - \overline{j} \Rightarrow AD = \|\overline{AD}\| = \sqrt{10}$ 2p (pentru D(3,0))

$\overline{BD} \overset{D-A}{=} 4\overline{i} - 3\overline{j} \Rightarrow BD = \|\overline{BD}\| = \sqrt{25}$ 2p

$\Rightarrow AB^2 + AD^2 = 5 + 10 = 15$, $BD^2 = 25$ $\Rightarrow AB^2 + AD^2 \neq BD^2$ 2p $\overset{RTP}{\Longrightarrow}$ $\triangle ABD$ nu e dreptunghic în A

$\Rightarrow m(A) \neq 90°$ 2p

2. $A(2,1)$, $B(1,3)$, $C(3,2)$

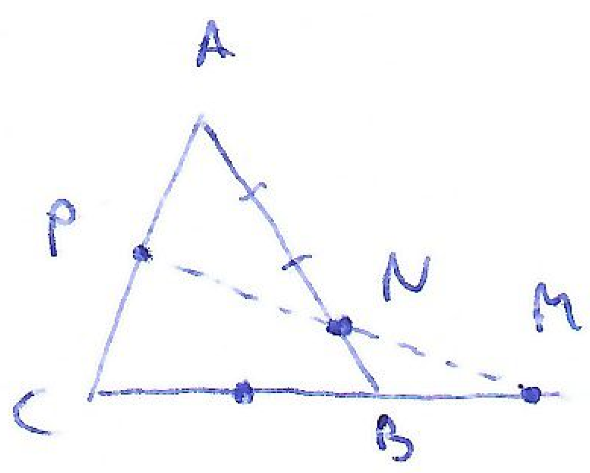
a) $\overline{AB} \overset{B-A}{=} -\overline{i} + 2\overline{j}$ 2p $\Rightarrow AB = \sqrt{5}$ 1p

$\overline{BC} \overset{C-B}{=} 2\overline{i} - \overline{j}$ 2p $\Rightarrow BC = \sqrt{5}$ 1p

$\overline{CA} \overset{A-C}{=} -\overline{i} - \overline{j}$ 2p $\Rightarrow AC = \sqrt{2}$ 1p

$\Rightarrow P_{ABC} = 2\sqrt{5} + \sqrt{2}$ 1p

b) $AC^2 = AB^2 + BC^2 - 2AB \cdot BC \cdot \cos B \overset{4p}{\Rightarrow} \cos B = \frac{AB^2 + BC^2 - AC^2}{2AB \cdot BC} = \frac{5+5-2}{2 \cdot 5} = \frac{8}{10} = \frac{4}{5}$ 6p

3. 

a) Th Menelaus $\Rightarrow \frac{MB}{MC} \cdot \frac{NA}{NB} \cdot \frac{PC}{PA} \overset{4p}{=} 1 \Rightarrow \frac{1}{3} \cdot 3 \cdot \frac{PC}{PA} = 1 \overset{2p}{\Rightarrow}$

$\Rightarrow \frac{PC}{PA} = 1 \overset{2p}{\Rightarrow} \frac{\overline{PC}}{\overline{PA}} = -1 \overset{2p}{\Rightarrow}$ P mijlocul seg. [AC]

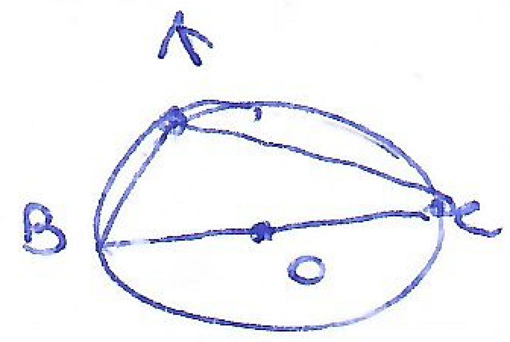
b) P mijloc ABC $\overset{4p}{\Rightarrow} A_{ABP} = A_{BPC} \overset{6p}{\Rightarrow}$ $\triangle ABP$ și $\triangle BPC$ sunt echivalente

(i.e. "au aceeași arie")

Secțiunea 7 - Vectori liberi în plan

Aprofundare of 10p

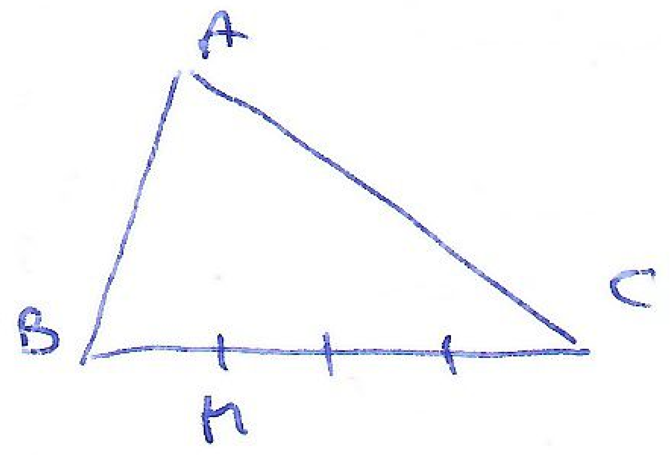
Partea I



1. $\overline{BO}=\overline{OC} \overset{2p}{\Rightarrow} BO \parallel OC \overset{2p}{\Rightarrow} O \in BC \overset{2p}{\Rightarrow} (BC)$ diametru

$\Rightarrow m(\widehat{BAC}) = \frac{1}{2} m(\widehat{BC}) = \frac{1}{2}\cdot 180^\circ = 90^\circ$ 2p $\Rightarrow$ $\triangle ABC$ dreptunghic în A 2p

2.

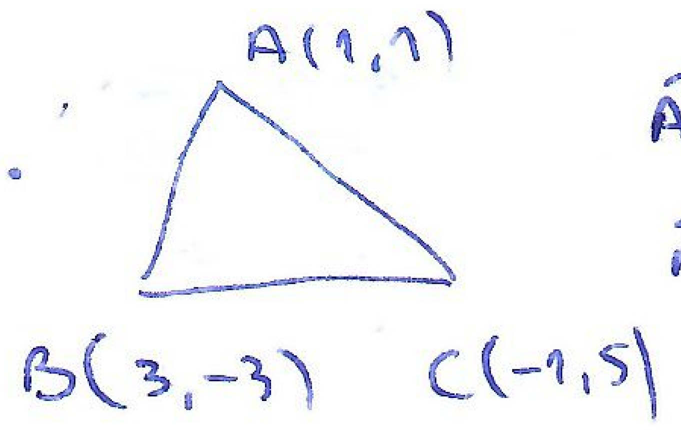


a) $\overline{AM} = \overline{AB} + \overline{BM} = \overline{AB} + \frac{1}{4}\overline{BC}$ 3p

Dar $\overline{BC} = \overline{BA} + \overline{AC} = -\overline{AB} + \overline{AC}$ 3p $\Rightarrow$

$\Rightarrow \overline{AM} = \overline{AB} + \frac{1}{4}\left(-\overline{AB} + \overline{AC}\right) = \frac{3}{4}\overline{AB} + \frac{1}{4}\overline{AC}$ 4p

3.



$\overline{AB} \overset{B-A}{=\!=} 2\bar{i} - 4\bar{j}$ 2p

$\overline{AC} \overset{C-A}{=\!=} -2\bar{i} + 4\bar{j}$ 2p

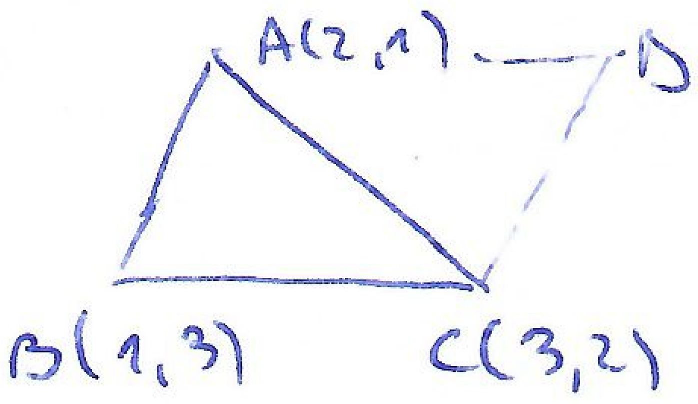
$\Rightarrow \overline{AB} = -\overline{AC} \overset{2p}{\Rightarrow} \overline{AB} \parallel \overline{AC} \overset{2p}{\Rightarrow}$ A, B, C coliniare 2p

Partea a II-a

1. a) $\bar{u} \parallel \bar{v} \overset{3p}{\Leftrightarrow} \frac{a-2}{4} = \frac{3}{10-a}$ CNN $\overset{3p}{\Rightarrow} a^2 - 12a + 32 = 0 < \begin{matrix}4\\8\end{matrix} \Rightarrow a \in \{4, 8\}$ 4p

b) $\bar{u} \perp \bar{v} \overset{3p}{\Leftrightarrow} \bar{u}\cdot\bar{v} = 0 \overset{3p}{\Leftrightarrow} (a-2)\cdot 4 + 3(10-a) = 0 \Leftrightarrow a = -22$ 4p

2.



a) $x_G = \frac{x_A + x_B + x_C}{3} = \frac{2+1+3}{3} = 2$ 5p

$y_G = \frac{y_A + y_B + y_C}{3} = \frac{1+3+2}{3} = 2$ 5p $\Rightarrow G(2,2)$

b) ABCD paralelogram $\overset{2p}{\Leftrightarrow} \overline{AD} = \overline{BC}$

pt $D(x,y)$ $\overline{AD} \overset{D-A}{=\!=} (x-2)\bar{i} + (y-1)\bar{j}$ 2p

iar $\overline{BC} \overset{C-B}{=\!=} 2\bar{i} - \bar{j}$ 2p

$\Rightarrow \begin{cases} x-2 = 2 & 2p \\ y-1 = -1 \end{cases} \Rightarrow \begin{cases} x = 0 \\ y = 0 \end{cases} \Rightarrow D(0,0)$ 2p

3. $2\overline{MN} = -3\overline{NP} \overset{2p}{\Leftrightarrow} 2\overline{NM} = 3\overline{NP} \overset{2p}{\Leftrightarrow} \frac{NM}{NP} = \frac{3}{2} \overset{2p}{\Rightarrow} \begin{cases} N \notin (MP) \\ NM = \frac{3}{2} NP \end{cases}$

a)

4p

b) $MP = \frac{1}{3} MN = 1$ 10p