Secțiunea 7 – Elemente de trigonometrie

Exersare of 10p

Partea I

1. $tg\,x + ctg\,x \overset{1p}{=} \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = \frac{\sin^2 x + \cos^2 x}{\sin x \cos x} \overset{2p}{=} \frac{1}{\sin x \cdot \cos x}$ $\Rightarrow tg\,x + ctg\,x = \frac{5}{2}$ 2p

$\sin^2 x + \cos^2 x = 1 \Rightarrow \cos^2 x = 1 - \frac{1}{5} \Rightarrow \cos x = \pm \frac{2}{\sqrt{5}}$ 2p

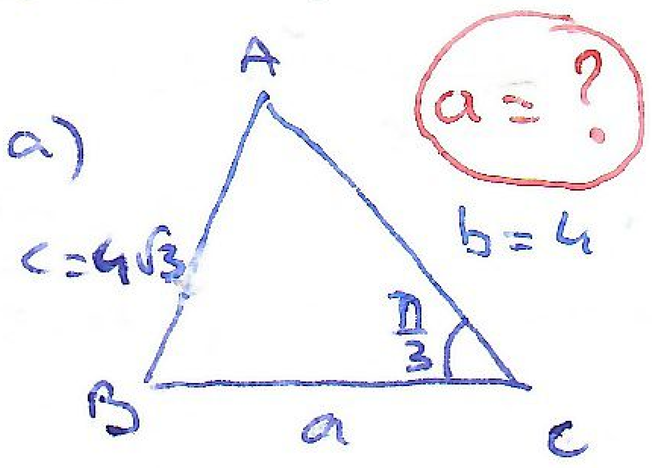
$x \in (0, \frac{\pi}{2}) \Rightarrow \cos x > 0$ 2p $\Rightarrow \cos x = \frac{2}{\sqrt{5}}$ 1p

2. $E(\frac{\pi}{3}) \overset{2p}{=} \cos\frac{\pi}{6} + \sin\frac{\pi}{3} \overset{3p}{=} 2\sin\frac{\pi}{3} \overset{3p}{=} 2\cdot\frac{\sqrt{3}}{2} \overset{2p}{=} \sqrt{3}$

3. $S_{ABC} \overset{4p}{=} \frac{AB\cdot AC\cdot \sin A}{2} = \frac{6\cdot 4\cdot \sin\frac{\pi}{6}}{2} = 3\cdot 4\cdot \frac{1}{2} = 6$ 6p

Partea a II-a

1. a)



a = ?

$c^2 = a^2 + b^2 - 2ab\cos C$

$\cos^2 C = 1 - \sin^2 C = \frac{1}{4} \Rightarrow \cos C = \pm\frac{1}{2}$ $\Rightarrow 48 = a^2 + 16 - 8a\cdot(\pm\frac{1}{2})$ 3p

Caz I: $a^2 - 4a - 32 = 0 < \begin{matrix}-4\\ 8\end{matrix} \Rightarrow \boxed{a=8} > 0$ 3p

Caz II: $a^2 + 4a - 32 = 0 < \begin{matrix}4\\ -8\end{matrix} \Rightarrow \boxed{a=4} > 0$ 3p

Așadar $(a,b,c) \in \{\underbrace{(8,4,4\sqrt{3})}_{4+4\sqrt{3}>8}, \underbrace{(4,4,4\sqrt{3})}_{4+4>4\sqrt{3}}\}$ 1p

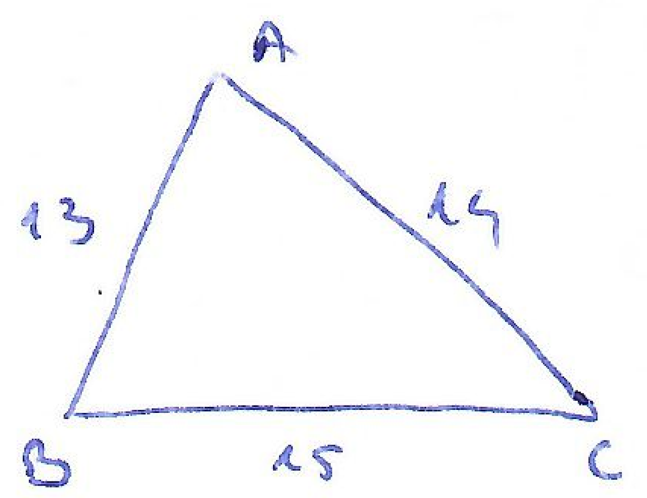
formează triunghi   formează triunghi

b) $b^2 = a^2 + c^2 - 2ac\cos B \Rightarrow \cos B = \frac{a^2+c^2-b^2}{2ac}$ 4p

pt $a = 4 \Rightarrow \cos B = \frac{4^2 + (4\sqrt{3})^2 - 4^2}{2\cdot 4\cdot 4\sqrt{3}} = \frac{1}{2\sqrt{3}} = \frac{\sqrt{3}}{6}$ 3p

pt $a = 8 \Rightarrow \cos B = \frac{8^2 + (4\sqrt{3})^2 - 4^2}{2\cdot 8\cdot 4\sqrt{3}} = \frac{6\cdot 4^2}{2\cdot 8\cdot 4\sqrt{3}} = \frac{\sqrt{3}}{2}$ 3p

2.



$S_{ABC} \overset{3p}{=} \sqrt{p(p-a)(p-b)(p-c)} = \sqrt{21\cdot 8\cdot 7\cdot 6} = 7\cdot 3\cdot 4 = 84\ cm^2$ 4p

$p = \frac{13+14+15}{2} = 21$ 3p

a) $S_{ABC} = \frac{h_A\cdot a}{2}$ 4p $\Rightarrow h_A = \frac{2S}{a} = \frac{2\cdot 84}{15} = \frac{168}{15}$ cm 2p

La fel $h_B = \frac{2S}{b} = \frac{168}{14} = \frac{84}{7}$ cm 2p și $h_C = \frac{2S}{c} = \frac{168}{13}$ cm 2p

b) $r \overset{4p}{=} \frac{S}{p} = \frac{84}{21} = 4$ cm 6p

3. a) $1 + tg^2 x \overset{3p}{=} 1 + \frac{\sin^2 x}{\cos^2 x} \overset{3p}{=} \frac{\cos^2 x + \sin^2 x}{\cos^2 x} \overset{4p}{=} \frac{1}{\cos^2 x}$, $\forall x \in (0, \frac{\pi}{2})$

b) $(1 + tg^2 x)\cos^2 x - (1 + ctg^2 x)\sin^2 x \overset{6p}{=} \frac{1}{\cos^2 x}\cdot\cos^2 x - \frac{1}{\sin^2 x}\cdot\sin^2 x \overset{4p}{=} 1 - 1 = 0$ $\forall x \in (0, \frac{\pi}{2})$

Secțiunea 7 – Elemente de trigonometrie

Aprofundare Of: 10p

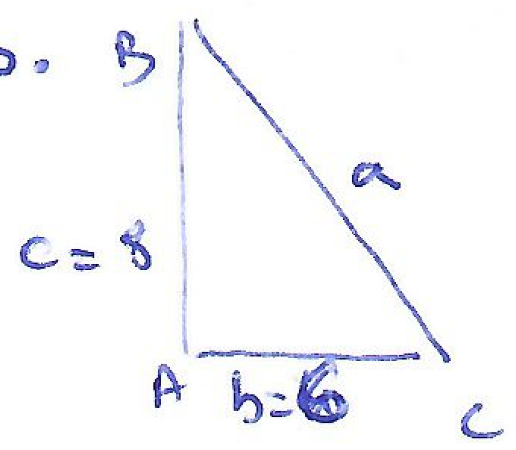
Partea I

1. $(\sin x+\cos x)^2 \overset{3p}{=} \sin^2 x + \underline{2\sin x\cos x} + \cos^2 x \overset{3p}{=} (\sin^2 x+\cos^2 x) + \underline{\sin 2x} = 1+\frac{1}{2} = \frac{3}{2}$ 4p

2. $\cos^2 x = \frac{1}{1+tg^2 x} = \frac{1}{1+1} = \frac{1}{2}$ 2p $\Rightarrow \sin^2 x = \frac{1}{2}$ 2p $\Rightarrow \sin x = \cos x = \frac{1}{\sqrt{2}}$ 2p $\Rightarrow$

cum $x \in (0, \frac{\pi}{2}) \Rightarrow \sin x > 0, \cos x > 0$ 2p

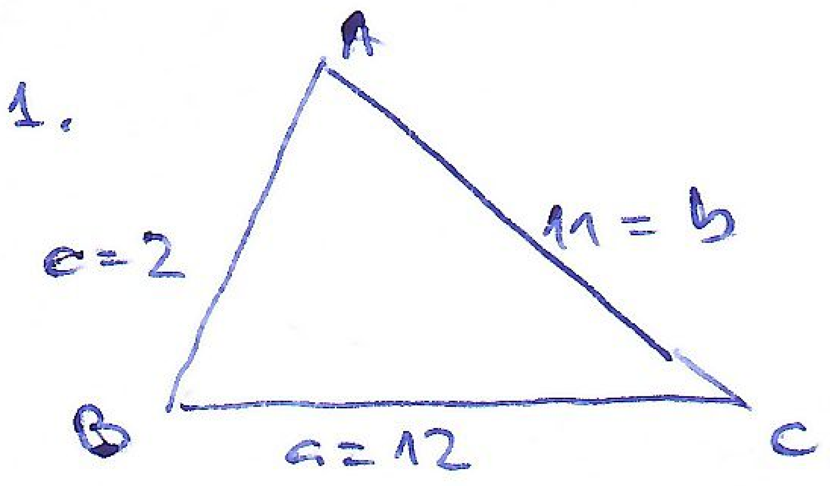
$\Rightarrow \sin x + 3\cos x = 4\cdot\frac{1}{\sqrt{2}} = 2\sqrt{2}$ 2p

3. 

$BC^2 = AB^2 + AC^2 \Rightarrow BC = 10$ cm 4p

Δ ABC dr. in A $\overset{3p}{\Rightarrow} R = \frac{ip}{2} = \frac{a}{2} = \frac{10}{2} = 5$ cm 3p

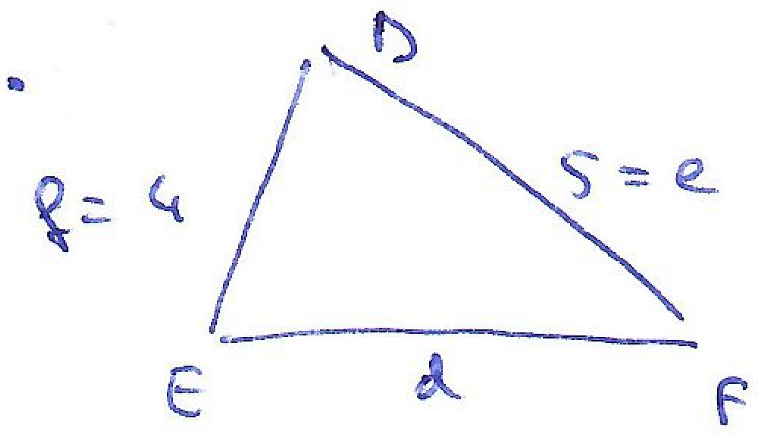
Partea a II-a

1. 

a) $a^2 = b^2+c^2-2bc\cos A \overset{3p}{\Rightarrow} \cos A = \frac{b^2+c^2-a^2}{2bc} = \frac{11^2+2^2-12^2}{2\cdot 11\cdot 2}$ 3p $\Rightarrow$

$\Rightarrow \cos A = -\frac{19}{44}$ 4p

b) $\cos A < 0 \overset{5p}{\Rightarrow} m(A) > 90^\circ \overset{5p}{\Rightarrow}$ Δ ABC obtuzunghic (in A)

2. 

a) $\frac{d+e+f}{2} \overset{5p}{=} p \Rightarrow d+5+4 = 2\cdot 6 \Rightarrow d = 3$ cm 5p

b) $3^2+4^2=5^2 \Rightarrow d^2+f^2=e^2 \Rightarrow$ Δ DEF dreptunghic in E 5p

$\Rightarrow S_{DEF} = \frac{d\cdot f}{2} = \frac{3\cdot 4}{2} = 6$ cm^2 5p

3. a) $\sin(\frac{\pi}{2}-x) \overset{5p}{=} \sin\frac{\pi}{2}\cos x - \sin x\cos\frac{\pi}{2} = 1\cdot\cos x - \sin x\cdot 0 = \cos x$ 5p $\forall x \in \mathbb{R}$

b) $\sin(\frac{\pi}{4}+x) - \cos(\frac{\pi}{4}-x) \overset{5p}{=} \sin(\frac{\pi}{4}+x) - \sin\left(\frac{\pi}{2}-(\frac{\pi}{4}-x)\right) =$

$\overset{3p}{=} \sin(\frac{\pi}{4}+x) - \sin(\frac{\pi}{4}+x) \overset{2p}{=} 0$