Sesiunea 6 – Funcția de gradul al II-lea

Exersare

Partea I: 10p

1. CE: $x+2 \neq 0 \Rightarrow x \neq -2$ 1p

Ecuația devine $\frac{2x^2+9x+8}{2(x+2)} = 2 \Leftrightarrow 2x^2+9x+8 = 4x+8$ 4p $\Leftrightarrow$

$\Leftrightarrow 2x^2+5x = 0 \Leftrightarrow 2x\left(x+\frac{5}{2}\right) = 0 \Leftrightarrow x \in \left\{0, -\frac{5}{2}\right\}$ 5p verifică CE.

$S = \left\{0, -\frac{5}{2}\right\}$

2. $(x+5)^2 - 3^2 > 0 \Leftrightarrow (x+5-3)(x+5+3) > 0 \Leftrightarrow (x+2)(x+8) > 0$ 5p $\Leftrightarrow$

$\Leftrightarrow x \in (-\infty, -8) \cup (-2, \infty)$ 5p

3. $f:\mathbb{R} \to \mathbb{R},\ f(x) = ax^2+bx+c,\ a \neq 0$

$A, B, C \in G_f \Rightarrow \begin{cases} f(0) = 1 \\ f(1) = 0 \\ f(2) = 3 \end{cases}$ 4p $\Leftrightarrow \begin{cases} c = 1 \\ a+b+c = 0 \\ 4a+2b+c = 3 \end{cases} \Leftrightarrow \begin{cases} c = 1 \\ a = 2 \\ b = -3 \end{cases}$ 4p

Așadar funcția este $f:\mathbb{R} \to \mathbb{R},\ f(x) = 2x^2-3x+1$ 2p

Partea a II-a:

1. a) $\Delta = 1 - 4\cdot(-4) = 17 > 0 \Rightarrow x_1 \neq x_2$ 5p
ecuația are coeficienți reali $\Rightarrow x_1 \neq x_2 \in \mathbb{R}$ 5p

b) RV: $\begin{cases} x_1+x_2 = -1 \\ x_1 \cdot x_2 = -4 \end{cases} \Rightarrow \begin{cases} S = -1 \\ P = -4 \end{cases}$ 4p

$\frac{x_1}{x_2} + \frac{x_2}{x_1} = \frac{x_1^2+x_2^2}{x_1 x_2} = \frac{S^2-2P}{P}$ 4p $= \frac{1+8}{-4} = -\frac{9}{4}$ 2p

2. a) $x^2+2x-14 < 0$

$x^2+2x-14 = 0 \Leftrightarrow x \in \{-1-\sqrt{15}, -1+\sqrt{15}\}$ 4p

$x^2+2x-14 < 0 \Leftrightarrow x \in (-1-\sqrt{15}, -1+\sqrt{15})$ 6p

b) Cum $3 = \sqrt{9} < \sqrt{15} < \sqrt{16} = 4 \Rightarrow 2 < -1+\sqrt{15} < 3$ 3p $\Rightarrow$

Din $-3 > -\sqrt{15} > -4 \Rightarrow -4 > -1-\sqrt{15} > -5$ 3p

$\Rightarrow -5 < -1-\sqrt{15} < -4 < 2 < -1+\sqrt{15} < 3 \Rightarrow$

$\Rightarrow \{n \in \mathbb{N} \mid n(n+2) < 14\} = \{0, 1, 2\}$ 3p $\Rightarrow$ Suma este $0+1+2 = 3$ 1p

3. a) $x_v = -\frac{b}{2a} = -\frac{3}{2} \Rightarrow$

x		$-\frac{3}{2}$	
x^2+3x+2	$\searrow$	y_v	$\nearrow$

4p

Cum $f \nearrow$ pe $\left(-\frac{3}{2}, \infty\right)$ și $-\frac{3}{2} < 1 < \frac{\pi}{3} < \sqrt{2}$ 2p $\Rightarrow f(1) < f\left(\frac{\pi}{3}\right) < f(\sqrt{2})$ 4p

b) $\text{Im} f = [y_v, \infty) = \left[-\frac{1}{4}, \infty\right)$ 5p

$-\frac{3}{2} < 0 < 3 \Rightarrow f((0, 3]) = (f(0), f(3)] = (2, 20]$ 5p

Secțiunea 6 - Funcția de gradul al II-lea

Aprofundare

Partea I:

1. CE: $x \neq -2$ 1p

Inecuația devine: $\frac{x+1}{x+2} + \frac{2x+3}{2} - 2 \leq 0 \Leftrightarrow \frac{x(2x+5)}{2(x+2)} \leq 0$ 4p $\Leftrightarrow x \in (-2, 0] \cup (-\infty, -\frac{5}{2}]$ 5p

2. $f: \mathbb{R} \to \mathbb{R}$, $f(x) = ax^2 + bx + c$, $a \neq 0$

A_g la Ox în $x_0 = 1 \Rightarrow \left.\begin{array}{l} f(1) = 0 \\ x_V = 1 \end{array}\right.$

G_f taie Oy în $y = -2 \Rightarrow f(0) = -2$ $\overset{4p}{\Rightarrow} \begin{cases} a+b+c=0 \\ -\frac{b}{2a} = 1 \\ c = -2 \end{cases} \Rightarrow \begin{cases} a = -2 \\ b = 4 \\ c = -2 \end{cases}$ 4p

$\Rightarrow f: \mathbb{R} \to \mathbb{R}$, $f(x) = -2x^2 + 4x - 2$ 2p

3. $x^2 + x + m < 0 \ \forall x \in \mathbb{R} \Rightarrow \Delta < 0 \overset{3p}{\Rightarrow} 1 - 4m < 0 \overset{3p}{\Rightarrow} m > \frac{1}{4} \Rightarrow m \in (\frac{1}{4}, \infty)$ 4p

Partea a II-a:

1. a) R.V. $\begin{cases} S = x_1 + x_2 = -\frac{b}{a} = -1 \\ P = x_1 \cdot x_2 = \frac{c}{a} = -5 \end{cases}$ 4p

$x_1^2 + x_1 - 5 = 0 \Longrightarrow x_1^3 + x_1^2 - 5x_1 = 0$

$x_2^2 + x_2 - 5 = 0 \Longrightarrow x_2^3 + x_2^2 - 5x_2 = 0$

$S_2 + S - 10 = 0 \Rightarrow S_2 = 10 - S = 11$ 3p $\quad S_3 + S_2 - 5S = 0 \Rightarrow S_3 = 5S - S_2 = -16$ 3p

b) $\frac{x_1}{x_2} + \frac{x_2}{x_1} = \frac{x_1^2 + x_2^2}{x_1 x_2} = \frac{S_2}{P} = \frac{-11}{5}$ 10p

2. a) $f(x) = 1 \Leftrightarrow x^2 - 2x + 1 = 0 \Leftrightarrow (x-1)^2 = 0 \Leftrightarrow x = 1$ 10p

b) $\operatorname{Im} f = \{y \in \mathbb{R} \mid \exists x \in \mathbb{R}$ a.î. $f(x) = y\}$ 2p

Fie $y \in \operatorname{Im} f \Rightarrow \exists x \in \mathbb{R}$ a.î. $\frac{2x}{x^2+1} = y \Rightarrow \exists x \in \mathbb{R}$ a.î. $yx^2 - 2x + y = 0$ 2p

PA $y = 0 \Rightarrow x = 0 \Rightarrow y = 0 \in \operatorname{Im} f$ 2p

PA $y \neq 0 \Rightarrow \Delta \geq 0$ 2p

$\Delta = (-2)^2 - 4y^2 = 4(1 - y^2)$ $\Rightarrow 1 - y^2 \geq 0 \Rightarrow y^2 \leq 1 \Rightarrow y \in [-1, 1]$ (lp)

Așadar $\operatorname{Im} f = [-1, 1]$ 2p

3. a) $\exists x_1 \neq x_2 \Rightarrow \Delta > 0$. 3p

$\Delta = (-m)^2 - 4(1-m) = m^2 + 4m - 4$ 3p $\Rightarrow m^2 + 4m - 4 > 0 \Leftrightarrow m \in (-\infty, -2-2\sqrt{2}) \cup (-2+2\sqrt{2}, \infty)$ 4p

b)

x		0		x_1		x_2	
$f(x)$			$+$	0	$-$	0	$+$

3p

$\begin{cases} \Delta > 0 \\ f(0) > 0 \\ 0 < x_V \end{cases} \overset{3p}{\Leftrightarrow} \begin{cases} m \in (-\infty, -2\sqrt{2}-2) \cup (-2+2\sqrt{2}; \infty) \\ 1 - m > 0 \\ 0 < -\frac{-m}{2} \end{cases}$ 2p $\Rightarrow \begin{cases} m \in (-\infty, -2-2\sqrt{2}) \cup (-2+2\sqrt{2}, \infty) \\ m < 1 \\ m > 0 \end{cases} \Rightarrow$

$\Rightarrow m \in (-2+2\sqrt{2}, 1)$ 2p