Secțiunea 5 - Șiruri de numere reale. Progresii aritmetice. Progresii geometrice

Exersare of: 10p

Partea I:

1. $a_1 = a_3 + (1-3)r$ 3p

$a_1 = 10 + (-2)\cdot 3 = 4$ 7p

2. $1-x,\ x,\ 2x+3$ $\div$ $\Leftrightarrow$ $x = \frac{(1-x)+(2x+3)}{2}$ 4p

$2x = 1-x+2x+3 \Leftrightarrow x = 4$ 6p

3. $2, 5, 8 \ldots 302$ sunt în progresie aritmetică cu $r = 5-2 = 3$ 3p

$a_1 = 2,\ a_n = 302$ 1p

$a_n = a_1 + (n-1)r \Rightarrow 302 = 2 + (n-1)\cdot 3 \Rightarrow n = 101$ 3p

$S = \frac{(a_1+a_n)\cdot n}{2} = \frac{(2+302)\cdot 101}{2} = 152\cdot 101 = 15200 + 152 = 15352$ 3p

Partea a II-a:

1. a) $a_2 + a_5 \overset{3p}{=} (a_1 + r) + (a_1 + 4r) = 2a_1 + 5r = 2a_1 + 10$ 3p

$a_2 + a_5 = 7 \Rightarrow 2a_1 + 10 = 7 \Rightarrow a_1 = -\frac{3}{2}$ 4p

b) $2+5 = 3+4 \Rightarrow a_3 + a_4 = a_2 + a_5 = 7$ 10p

2. a) $a_n \overset{5p}{=} (a_1 + a_2 + \ldots + a_{n-1} + a_n) - (a_1 + a_2 + \ldots + a_{n-1}) =$

$= S_n - S_{n-1} = (2^n + 1) - (2^{n-1} + 1) = 2^{n-1}$ 5p, $\forall n \geq 2$

b) pentru $n \geq 2$ $a_n = 2^{n-1}$

atunci $\frac{a_{n+1}}{a_n} = \frac{2^{n+1-1}}{2^{n-1}} = 2$ constant 5p $\forall n \geq 2 \Rightarrow$

$\Rightarrow (a_n)_{n\geq 2}$ $\div\div$ cu rația $q = 2$ 5p

3. a) $S = \underset{a_1}{10} + \underset{a_2}{10^2} + \ldots + \underset{a_{10}}{10^{10}} \overset{3p}{=} a_1 \cdot \frac{2^{10}-1}{2-1} \overset{3p}{=} 10\cdot\frac{10^{10}-1}{10-1} = 10\cdot\frac{\overbrace{99\ldots9}^{10 \text{ cifre}}}{9}$ 3p

$S = \underbrace{11\ldots1}_{10 \text{ cifre}}0$ 1p

b) $T = 9 + 99 + \ldots + \underbrace{99\ldots9}_{10 \text{ cifre}} \overset{5p}{=} (10-1) + (10^2-1) + \ldots + (10^{10}-1) =$

$\overset{3p}{=} (10 + 10^2 + \ldots + 10^{10}) - 10 \overset{a)}{=} \underbrace{11\ldots1}_{10 \text{ cifre}}0 - 10 = \underbrace{11\ldots1}_{9 \text{ cifre}}00$ 2p

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Aprofundare

of: 10 p

Partea I

1. $b_1 \cdot b_2 \cdot b_3 \overset{5p}{=} \frac{b_2}{2} \cdot b_2 \cdot (b_2 2) \overset{3p}{=} b_2^3 = 4^3 = 2^6 = 64$ 2p

2. $\underbrace{1 + 6 + 11 + \dots}_{} + x = 1071$

Observ că termenii sunt în progresie aritmetică cu $r = 5$ 2p

$a_1 = 1, \quad a_n = x$

Dar $a_n = a_1 + (n-1) r \Rightarrow x = 1 + (n-1) \cdot 5 \Rightarrow n = \frac{x-1}{5} + 1 = \frac{x+4}{5}$ 2p

$MS = \frac{(a_1 + a_n) \cdot n}{2} = \frac{(1+x) \cdot \frac{x+4}{5}}{2} = \frac{(x+1)(x+4)}{10}$ 2p

$\frac{(x+1)(x+4)}{10} = 1071 \Rightarrow x \in \{-106, 101\}$ 2p

Dar $x > 0 \Rightarrow x = 101$ 2p

3. $2, a, b \div \overset{2p}{\Leftrightarrow} a = \frac{b+2}{2}$

$2, 17, a \div\div \overset{2p}{\Leftrightarrow} 17^2 = 2a \Rightarrow a = \frac{17^2}{2} = \frac{289}{2}$ 3p

Cum $a = \frac{b+2}{2} \Rightarrow b = 2a - 2 = 287$ 3p

Soluție: $(a, b) = \left(\frac{289}{2}, 287\right)$

Partea a II-a:

1. a) $a_2 = a_1 + r \Rightarrow r = a_2 - a_1 = 7 - 4 = 3$ 5p

$a_3 = a_1 + 2r = 4 + 2 \cdot 3 = 10$ 5p

b) $S_{100} = a_1 + a_2 + \dots + a_{100} = \frac{(a_1 + a_{100}) \cdot 100}{2}$ 3p

$a_{100} = a_1 + (100-1) r = 4 + 99 \cdot 3 = 300 + 1 = 301$ 3p

$S_{100} = \frac{(4 + 301) \cdot 100}{2} = \frac{30500}{2} = 15250$ 4p

2. a) $S_n - S_{n-1} \overset{5p}{=} (n^2 + 2n) - [(n-1)^2 + 2(n-1)] = 2n + 1 \quad \forall n \geq 2$ 5p

b) pt $n \geq 2$ $\quad a_n = S_n - S_{n-1} = 2n+1$ 4p

pt $n \geq 2$ $\quad a_{n+1} - a_n = [2(n+1) + 1] - (2n+1) = 2$ constant $\overset{3p}{\Rightarrow}$

$\Rightarrow (a_n)_{n \geq 2} \div$ cu $r = 2$ 3p

3. a) $5 + 16 = 1 + 20 \overset{5p}{\Rightarrow} a_1 + a_{20} = a_5 + a_{16} \Rightarrow (a_5 + a_{16}) - (a_1 + a_{20}) = 0$ 5p

b) $S_{20} = a_1 + a_2 + \dots + a_{20} \overset{4p}{=} \frac{(a_1 + a_{20}) \cdot 20}{2} \overset{4p}{=} \frac{(a_5 + a_{16}) \cdot 20}{2} = \frac{4 \cdot 20}{2} = 40$ 2p