

Section 4 - Functii de gradul I

Exersare Of: 10 P

Parte I

1. CE: $x \neq -1$ 5P

$$x^2 + 2x + 3 = x(x+1) \Leftrightarrow x = -3 \neq -1 \text{ solutie } 4P$$

$$S = \{-3\} \text{ 1P}$$

2. CE: $x \neq -1$ 3P

$$\frac{x^2 + 2x + 3}{x+1} \leq x \Leftrightarrow \frac{x+3}{x+1} \leq 0 \text{ 3P}$$

| | | | | |
|-------------------|--|----|----|---|
| x | | -3 | -1 | |
| $x+1$ | | - | - | 0 |
| $x+3$ | | - | 0 | + |
| $\frac{x+3}{x+1}$ | | + | 0 | - |

$$\Rightarrow x \in [-3, -1) \text{ 3P}$$

$$S = \{-3, -1) \text{ 1P}$$

3. $\begin{cases} x+y=3 \\ x-y=1 \end{cases} \xrightarrow{\text{①}+\text{②}} \begin{cases} 2x=4 \\ 2y=2 \end{cases} \Leftrightarrow \begin{cases} x=2 \\ y=1 \end{cases} \Rightarrow S = \{(2,1)\} \text{ 2P}$

Parte II

1. a) $f(0) = 2 \Rightarrow d \cap O_y = \{A(0,2)\} \text{ 5P}$

$$f(x) = 0 \Leftrightarrow 3x+2=0 \Leftrightarrow x = -\frac{2}{3} \Rightarrow d \cap O_x = \{B(-\frac{2}{3}, 0)\} \text{ 5P}$$

b) $A(10,y) \in d \Rightarrow f(10) = y \Rightarrow y = 32 \text{ 10P}$

2. a) $f \nearrow \Leftrightarrow 3-m^2 > 0 \Leftrightarrow m \in (-\sqrt{3}, \sqrt{3}) \text{ 10P}$

b) $3-m^2 = 0 \Rightarrow m = \pm\sqrt{3} \text{ 5P}$

$$\downarrow$$

$$f(x) = m > 0 \forall x \Rightarrow \boxed{m = \sqrt{3}} \text{ 4P}$$

$$S = \{\sqrt{3}\} \text{ 1P}$$

3. a) $f \nearrow \Rightarrow a > 0 \text{ (*) } \text{ 2P}$

$$f(f(x)) = a f(x) + b = a(ax+b) + b = a^2x + ab + b \text{ 3P}$$

$$f(f(x)) = 4x+3 \Leftrightarrow \begin{cases} a^2 = 4 \\ ab+b = 3 \end{cases} \Rightarrow a = \pm 2 \text{ (*) } a = 2 \Rightarrow b = 1 \text{ 1P}$$

$$S = \{(2,1)\} \text{ 1P}$$

b) pt $a=2, b=1 \Rightarrow f(x) = 2x+1 \nearrow \text{ pe } \mathbb{R} \Rightarrow f([0,2]) = (f(0), f(2)) = (1,5) \text{ 5P}$

Secțiunea 4 - Funcții de gradul I

Aprofundare **0810P**

Parte I

1. CE: $x \neq -3$ **5P**

$$(x+1)(x+2) = (x+3)(x+4) \Leftrightarrow 3x+2 = 7x+10 \Leftrightarrow x = -2 \neq -3 \text{ soluție } \mathbf{4P}$$

$$S = \{-2\} \mathbf{1P}$$

2. CE: $x \neq -3, x \neq -4$ **3P**

$$\frac{x(x+1)}{(x+2)(x+3)} \geq 1 \Leftrightarrow \frac{x^2+x-(x^2+5x+6)}{x(x+3)} \geq 0 \Leftrightarrow \frac{4x+6}{x(x+3)} \leq 0 \Leftrightarrow \frac{2x+3}{x(x+3)} \leq 0 \mathbf{3P}$$

| x | -3 | $-\frac{3}{2}$ | 0 | |
|-----------------------|------|----------------|-----|-------|
| $2x+3$ | - | - | 0 | + + |
| x | - | - | - | 0 + |
| $x+3$ | - | 0 | + | + - |
| $\frac{2x+3}{x(x+3)}$ | - | | + 0 | - + |

$= x \in (-\infty, -3) \cup [-\frac{3}{2}, 0) \mathbf{3P}$

$$S = (-\infty, -3) \cup [-\frac{3}{2}, 0) \mathbf{1P}$$

3.
$$\begin{cases} x+y+z=3 & \textcircled{1} \\ x+y-z=1 & \textcircled{2} \\ x-y+z=1 & \textcircled{3} \end{cases} \xrightarrow{\textcircled{1}+\textcircled{2}} \begin{cases} x+y+z=3 \\ 2z=2 \\ 2y=2 \end{cases} \Leftrightarrow \begin{cases} x=1 \\ y=1 \\ z=1 \end{cases} \Rightarrow S = \{(1, 1, 1)\} \mathbf{4P}$$

Parte II

1. a) $f(x) < 8 \Leftrightarrow 3x+2 < 8 \Leftrightarrow 3x < 6 \Leftrightarrow x < 2 \Rightarrow S = \{0, 2\} \mathbf{5P}$

b) $3 > 0 \Rightarrow f(x) \nearrow \Rightarrow f([0, 4]) = [f(0), f(4)] = [2, 14] \mathbf{2P}$

$$f(-\infty, 1) = (-\infty, f(1)) = (-\infty, 5) \mathbf{2P}$$

2. a) $f(0) = 1 \Leftrightarrow a+b = 1 \text{ (1)} \mathbf{2P}$

$$f(x+1) = a(x+1)+b = ax+a+b \Rightarrow$$

$$\Rightarrow f(x+1) = f(x+2) \Leftrightarrow ax+a+b = ax+b+2 \Rightarrow \boxed{a=2} \Leftrightarrow \boxed{b=-1} \mathbf{2P}$$

$$\Rightarrow S = \{(2, -1)\} \mathbf{2P}$$

b) pentru $a=2, b=-1 \Rightarrow f(x) = 2x-1 \nearrow \text{ pe } \mathbb{R} \mathbf{2P}$

$$\Rightarrow f([0, 4]) = [f(0), f(4)] = [-1, 7] \mathbf{2P}$$

$$f(-\infty, 1) = (-\infty, f(1)) = (-\infty, 1) \mathbf{2P}$$

3. a) $f \nearrow \text{ pe } \mathbb{R} \Rightarrow m^2-1 > 0 \Rightarrow m \in (-\infty, -1) \cup (1, \infty) \mathbf{10P}$

b) $S = \sum_{k=1}^{2015} (3k+1) = 3(1+3+5+\dots+2015) + 1010 \stackrel{4P}{=} 3 \cdot 1010^2 + 2010 = 3061310 \mathbf{1P}$

$$1+3+\dots+2015 = \sum_{k=1}^{2015} 2k-1 = 2 \sum_{k=1}^{1007} k - 1010 = 2 \cdot \frac{1007 \cdot 1008}{2} - 1010 = 1010^2 \mathbf{5P}$$