Secțiunea 3 - Funcții numerice

Exersare of: 10p

Partea I

1. $f(x)=g(x) \Leftrightarrow x^2+2x+3=x+5 \Leftrightarrow x^2+x-2=0 < \begin{matrix} 1 \\ -2 \end{matrix}$ 3p

pt $x=-2 \Rightarrow g(-2)=-2+5=3 \Rightarrow A(-2,3) \in G_f \cap G_g$ 3p

pt $x=1 \Rightarrow g(1)=1+5=6 \Rightarrow A(1,6) \in G_f \cap G_g$ 3p

$S=\{A(-2,3), B(1,6)\}$ 1p

2. $f(1)=1^2-2\cdot 1+4=3 \Rightarrow A(1,3) \in G_f$ 10p

3. $f(x)=x+1$

$f\circ f(x) = f(f(x)) = f(x)+1 = x+1+1 = x+2$ 3p

$f\circ f\circ f(x) = f(f\circ f(x)) = f\circ f(x)+1 = x+2+1 = x+3$ 3p

Ecuația devine $x+3=3 \Rightarrow x=3 \in D_f=\mathbb{R}$ 4p

Partea II

1. a) $f\circ f(1) \overset{3p}{=} f(f(1)) \overset{3p}{=} 2f(1)+1 \overset{3p}{=} 2\{2\cdot 1+1\}+1=7$ 1p

b) $\sum_{k=1}^{100} f(k) = \sum_{k=1}^{100}(2k+1) \overset{6p}{=} 2\sum_{k=1}^{100} k + 100 \overset{3p}{=} 2\cdot\frac{100(100+1)}{2} + 100 = 10200$ 1p

2. a) $f\circ g(1) \overset{3p}{=} f(g(1)) \overset{3p}{=} g(1)-2 = (2-1)-2 = -1$ 4p

b) $\sum_{k=1}^{100}\left[(f\circ g)(k)\right]^2 = \sum_{k=1}^{100}(-k)^2 = \sum_{k=1}^{100} k^2 \overset{3p}{=} \frac{100(101)(201)}{6} = 338.350$ 2p

$f\circ g(k) = f(g(k)) = g(k)-2 = (2-k)-2 = -k$ 5p

3. a)

x	$x_v = \frac{-b}{2a} = -\frac{3}{2}$
x^2+3x+2	$\searrow$ y_v $\nearrow$

5p $\Rightarrow$

$\Rightarrow g: \mathbb{R}\to\mathbb{R}, g(x)=x^2+3x+2 \nearrow$ pe $[-\frac{3}{2}, \infty)$

Cum $f = g|_{[0,\infty)} \Rightarrow [0,\infty) \subset [-\frac{3}{2}, \infty)$ $\Rightarrow f \nearrow$ deci mono 5p

b) $f\nearrow$ pe $[0,\infty) \Rightarrow f([0,4)) \overset{5p}{=} [f(0), f(4)) = [2,30)$ 5p

Secțiunea 3 – Funcții numerice. Proprietăți (I)

Aprofundare Of 10p

Partea I

1. $f(1)=g(1) \Leftrightarrow 1^2+2\cdot1+3=1+a \Leftrightarrow \boxed{a=5}$ 10p

2. $A(1,3)\in G_f \overset{5p}{\Rightarrow} f(1)=3 \Rightarrow 1^2-m\cdot1+2m=3 \Rightarrow \boxed{m=2}$ 5p

3. $f\circ f(x)=f(f(x))=f(x)+1=(x+1)+1=x+2$ 3p

$f^2(x)=(f(x))^2=(x+1)^2=x^2+2x+1$ 3p

Ecuația devine:

$x^2+2x+1=x+2 \Leftrightarrow x^2+x-1=0 \Leftrightarrow x=\frac{-1\pm\sqrt5}{2}$ 3p

$S=\left\{\frac{-1\pm\sqrt5}{2}\right\}$ 1p

Partea II

1. a) $f(1)=1^2-2\cdot1+1=0$ 3p

$(f\circ f)(1)\overset{3p}{=}f(f(1))=f(0)=0^2-2\cdot0+1=1$ 4p

b) $\sum_{k=1}^{100} f(k)=\sum_{k=1}^{100}(k^2-2k+1)\overset{3p}{=}\sum_{k=1}^{100}k^2-2\cdot\sum_{k=1}^{100}k+100=$

$\overset{6p}{=}\frac{100(100+1)\cdot(2\cdot100+1)}{6}-2\cdot\frac{100(100+1)}{2}+100=$

$=\frac{100\cdot101\cdot201}{6}-10100+100=328.350$ 1p

2. a) $g(1)=2-1=1$ 3p

$(f\circ g)(1)\overset{3p}{=}f(g(1))=f(1)=(1-1)^2=0$ 4p

b) $\sum_{k=1}^{100}(f\circ g)(k)=\sum_{k=1}^{100}k^2-2\cdot k+1=328.350$ 5p

$(f\circ g)(k)=f(g(k))=(g(k)-1)^2=(1-k)^2=k^2-2k+1$ 5p

3. a) $x+1\nearrow$ pe $[0,\infty)\Rightarrow\frac{1}{x+1}\searrow$ pe $[0,\infty)$ 10p

b) $f\searrow$ pe $[0,\infty)\Rightarrow f([0,4))\overset{5p}{=}(f(4),f(0)]=\left(\frac15,1\right]$ 5p