Secțiunea 2 - Numere reale

Exersare Of: 10p

Parte I

1. $[-\pi] = -4$ 3p

$[\sqrt{2}] = 1$ 3p

$[-\pi] + [\sqrt{2}] = -4 + 1 = -3$ 4p

2. $[x] = t$

$t^2 - 3t + 2 = 0$ are rădăcinile $t_1 = 1$, $t_2 = 2$ 3p

$[x] = 1 \Rightarrow x \in [1, 2)$ 3p

$[x] = 2 \Rightarrow x \in [2, 3)$ 3p

$S = [1, 2) \cup [2, 3) = [1, 3)$ 1p

3. $|1 - 2x| > 1 \Rightarrow 1 - 2x > 1$ sau $1 - 2x < -1$ 3p

$1 - 2x > 1 \Rightarrow 2x < 0 \Rightarrow x \in (-\infty, 0)$ 3p

$1 - 2x < -1 \Rightarrow 2x > 2 \Rightarrow x \in (1, \infty)$ 3p

$S = (-\infty, 0) \cup (1, \infty)$ 1p

Parte II

1. a) $a = \frac{3}{22} = 0,1(36)$ 3p

$a_0 = 0, \quad a_1 = 1, \quad a_k = \begin{cases} 3, & k = 2p \\ 6, & k = 2p+1 \end{cases}$ pt $p \geq 1$ 4p

$a_{10} = a_{2\cdot 5} = 3$ 3p

b) $\sum_{k=1}^{100} a_k \overset{3p}{=} a_1 + (a_2+a_3) + (a_4+a_5) + \ldots + (a_{98}+a_{99}) + a_{100} =$

$\overset{3p}{=} 1 + \underbrace{(3+6) + (3+6) + \ldots + (3+6)}_{\frac{99-2+1}{2} = \frac{98}{2} = 49 \text{ perechi}} + 3 =$

$\overset{3p}{=} 1 + 9\cdot 49 + 3 \overset{1p}{=} 445$

2. a) $\frac{\overset{k+1}{\cancel{1}}}{k} - \frac{\overset{k}{\cancel{1}}}{k+1} = \frac{k+1-k}{k(k+1)} = \frac{1}{k^2+k} \Rightarrow a_k = \frac{1}{k} - \frac{1}{k+1}$ 10p

b) $\sum_{k=1}^{100} a_k = a_1 + a_2 + \ldots + a_{99} + a_{100} =$

$\overset{5p}{=} \left(\frac{1}{1} - \cancel{\frac{1}{2}}\right) + \left(\cancel{\frac{1}{2}} - \cancel{\frac{1}{3}}\right) + \ldots + \left(\cancel{\frac{1}{99}} - \cancel{\frac{1}{100}}\right) + \left(\cancel{\frac{1}{100}} - \frac{1}{101}\right)$

$= 1 - \frac{1}{101} \overset{5p}{=} \frac{100}{101}$

3. a) cazuri posibile: 200 3p

cazuri favorabile: $1^2, 2^2, \ldots, 14^2 = 196 \Rightarrow 14$ (deoarece $15^2 = 225 > 200$) 3p

$p(A) = \frac{14}{200} = \frac{7}{100}$ ($= 0,07$ sau 7%) 4p

b) $k^2 \vdots 15$, $15 = 3\cdot 5$, $(3,5) = 1$ $\overset{5p}{\Rightarrow} k \vdots 3$ și $k \vdots 5 \Rightarrow k \vdots 15$

Dar $k \in \{1, 2, \ldots, 14\}$ $\overset{3p}{\Rightarrow}$ nu există pătrate perfecte divizibile la 15 în N

$\Rightarrow$ cazuri favorabile: 0 $\Rightarrow$ $p(B) = 0$ 2p

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Aprofundare

Parte I

1. $\frac{1}{\sqrt{2}-1} = \frac{\sqrt{2}+1}{\sqrt{2}^2-1} = \sqrt{2}+1$ 3p

$1 \le \sqrt{2} < 2 \overset{3p}{\Rightarrow} 2 \le \sqrt{2}+1 < 3 \overset{3p}{\Rightarrow} [\sqrt{2}+1] = 2$

$a = \left[\frac{1}{\sqrt{2}-1}\right] = [\sqrt{2}+1] = 2$ 1p

2. $|x+2| + |x-1| = |x+2| + |1-x| \overset{3p}{\geq} |x+2+1-x| = 3$

Egalitatea are loc $\overset{3p}{\Leftrightarrow} (x+2)(1-x) \geq 0 \overset{3p}{\Leftrightarrow} x \in [-2, 1]$

$S = [-2, 1]$ 1p

3. $\left[\frac{3x+1}{2}\right] + \left[\frac{3x+2}{2}\right] = \left[\frac{3x+1}{2}\right] + \left[\frac{3x+1}{2} + \frac{1}{2}\right] \overset{\text{Hermite}}{\underset{3p}{=}} 2\left[\frac{3x+1}{2}\right]$

Ecuația devine

$2\left[\frac{3x+1}{2}\right] \geq 1 \Leftrightarrow \left[\frac{3x+1}{2}\right] \geq \frac{1}{2} \Leftrightarrow \left[\frac{3x+1}{2}\right] \overset{3p}{\geq} 1 \Leftrightarrow \frac{3x+1}{2} \geq 1 \Leftrightarrow$

$\Leftrightarrow 3x+1 \geq 2 \Leftrightarrow x \geq \frac{1}{3}$ 3p

$S = [\frac{1}{3}, \infty)$ 1p

Parte II

1. a) $a_k = \frac{1}{\sqrt{k}+\sqrt{k+1}} \overset{3p}{=} \frac{\sqrt{k+1}-\sqrt{k}}{(\sqrt{k+1})^2-\sqrt{k}^2} \overset{3p}{=} \sqrt{k+1} - \sqrt{k} \quad \forall k \geq 1$ 4p

b) $\sum_{k=1}^{99} a_k = a_1 + a_2 + \ldots + a_{98} + a_{99} \overset{\text{not}}{=} S$

Dar $a_1 = \sqrt{2} - \sqrt{1}$

$a_2 = \sqrt{3} - \sqrt{2}$

...

$a_{98} = \sqrt{99} - \sqrt{98}$

$a_{99} = \sqrt{100} - \sqrt{99}$ 3p

$S \overset{3p}{=} \sqrt{100} - \sqrt{1} = 10 - 1 = 9$ 4p

2. a) $\forall k \geq 3 \quad k^2+2k+1 \leq k^2+3k+1 < k^2+4k+4$ 3p

$\Rightarrow \forall k \geq 3 \quad (k+1)^2 \leq k^2+3k+1 < (k+2)^2$

$\Rightarrow \forall k \geq 3 \quad k+1 \leq \sqrt{k^2+3k+1} < k+2$ 3p

$\Rightarrow \forall k \geq 3 \quad a_k = \{\sqrt{k^2+3k+1}\} = k+1$ 4p

b) $\sum_{k=3}^{100} a_k = \sum_{k=3}^{100} (k+1) = 4+5+\ldots+101 =$

3p $= 1+2+3+4+5+\ldots+101 - (1+2+3) =$

$= \frac{101 \cdot 102}{2} - 6 = 5145$ 7p

3. a)

x	0	1	2
$f(x)$	$f(0)$	$f(1)$	$f(2)$

3p (2 moduri) ($f(0) \in \{0, 2\}$)

(3 moduri) 2p

(3 moduri) 2p

$\Rightarrow 2 \cdot 3 \cdot 3 = 18$ funcții cu proprietatea că $f(0)$ e par 3p

b) $f(0)+f(2) = 2 \Rightarrow (f(0), f(2)) \in \{(0,2), (1,1), (2,0)\}$ 3p

x	1	0	2
$f(x)$	$f(1)$	$f(0)$	$f(2)$

$\underbrace{f(0) \quad f(2)}_{3 \text{ moduri}}$

3 moduri 3p

$\Rightarrow 3 \cdot 3 = 9$ funcții 4p