Comparați:

a) $\int_1^4 \ln x \, dx$ și $\int_1^4 \frac{x-1}{x} \, dx$

$\ln x \boxed{>} \frac{x-1}{x} \overset{|\cdot x>1>0}{\iff} x\ln x \boxed{>} x-1 \iff \underbrace{x\ln x - x + 1}_{f(x)} \boxed{>} 0$ pt că:

$f'(x) = \ln x + 1 - 1 = \ln x \geq 0 \quad \forall x > 1$

x		1		4	
$f'(x)$	//	[	$+$	]	//
$f(x)$	//	[	$\nearrow$	]	//

$\Rightarrow f(x) > f(1) = 0 \quad \forall x \in [1,4]$

Așadar

$\ln x > \frac{x-1}{x} \quad \forall x \in [1,4] \Rightarrow \int_1^4 \ln x \, dx > \int_1^4 \frac{x-1}{x} \, dx$

b) $\int_{\sqrt{2}}^{\sqrt{3}} x \,\mathrm{arctg}\, x \, dx$? $\int_{\sqrt{2}}^{\sqrt{3}} \ln(1+x^2) \, dx$

$x \,\mathrm{arctg}\, x \ \square \ \ln(1+x^2) \iff \underbrace{x \,\mathrm{arctg}\, x - \ln(1+x^2)}_{f} \ \square \ 0$ (*)

$f : \mathbb{R} \to \mathbb{R}$

$f' = \mathrm{arctg}\, x + x \cdot \frac{1}{1+x^2} - \frac{2x}{1+x^2} = \mathrm{arctg}\, x - \frac{x}{1+x^2}$

$f'' = \frac{1}{1+x^2} - \frac{1+x^2-2x^2}{(1+x^2)^2} = \frac{2x^2}{(1+x^2)^2}$

x		0		$\sqrt{2}$	$\sqrt{3}$
$f''(x)$ semn	$+$	0	$+$		
$f'(x)$ monot	$\nearrow$		$\nearrow$		
$f'(x)$ semn	$-$	0	$+$	$+$	
f monot	$\searrow$		$\nearrow$		

$\Rightarrow f(x) > f(\sqrt{2}) \quad \forall x \in [\sqrt{2}, \sqrt{3}]$

$f(\sqrt{2}) = \underbrace{\sqrt{2}\,\mathrm{arctg}\,\sqrt{2} - \ln 3}$

POZITIV ? NEGATIV?

Folosesc $f \nearrow$ pe $[0, \infty) \Rightarrow$

$\Rightarrow f(x) > f(0) = 0 \quad \forall x \in [0, \infty) \Rightarrow$

$\Rightarrow f(x) > 0 \quad \forall x \in [\sqrt{2}, \sqrt{3}] \overset{(*)}{\Rightarrow}$

$\Rightarrow x \,\mathrm{arctg}\, x > \ln(1+x^2) \quad \forall x \in [\sqrt{2}, \sqrt{3}] \Rightarrow$

$\Rightarrow \int_{\sqrt{2}}^{\sqrt{3}} x \,\mathrm{arctg}\, x \, dx > \int_{\sqrt{2}}^{\sqrt{3}} \ln(1+x^2) \, dx$

Demonstrați inegalitățile:

a) $\int_0^{e^2} \ln(5x+4)\,dx \le \int_0^{e^2} (5x+3)\,dx$

rez

Met I

$\ln(5x+4) \underset{?}{\le} 5x+3 \quad \forall x \in [0, e^2]$?

$\underbrace{\ln(5x+4) - 5x+3}_{f(x)} < 0 \quad \forall x \in [0, e^2]$

$f:[0, e^2] \to \mathbb{R}$

$f' = \frac{5}{5x+4} - 5 = 5\left(\frac{1}{5x+4} - 1\right) = 5 \cdot \frac{-5x-3}{5x+4} = -5 \cdot \frac{5x+3}{5x+4} < 0 \quad \forall x \ge 0$

x	0		e^2
$f'(x)$	[	$-$	]
$f(x)$	[	$\searrow$	]

$\Rightarrow f(x) \le f(0) = \ln 2 - 3 < 1 - 3 < 0$ adev.

$\Rightarrow \ln(5x+4) < 5x+3 \quad , \forall x \in [0, e^2] \Rightarrow$

$\Rightarrow \int_0^{e^2} \ln(5x+4)\,dx < \int_0^{e^2} 5x+3\,dx$

Met II

Știm că $\boxed{\ln(1+x) < x \quad \forall x \in (0, \infty)}$

Atunci $\ln(1 + 5x+3) < 5x+3 \quad \forall x \in (0, e^2)$ (pt că $5x+3 > 0$)

$\Rightarrow \int_0^{e^2} \ln(5x+4)\,dx \le \int_0^{e^2} 5x+3\,dx \Rightarrow$

$\Rightarrow \int_0^{e^2} \ln(5x+4)\,dx \le \int_0^{e^2} 5x+3\,dx$

b) $\int_2^3 e^{x^2}\,dx \geq \int_2^3 x^2+1\,dx$

sol

Dem că $e^{x^2} \geq x^2+1 \quad \forall\, x \in [2,3]$ (1) $\Leftarrow$ $e^{y} \geq y+1 \quad \forall y \in [4,9]$

Dem că $e^{x} > x+1 \quad \forall\, x \in [4,9]$ (2)

$\Updownarrow$

$\underbrace{e^{x} - x - 1}_{f(x)} > 0 \quad \forall\, x \in [4,9]$, $f: [4,9] \to \mathbb{R}$

$f' = e^{x} - 1 \quad \forall\, x \in [4,9]$, $f' : [4,9] \to \mathbb{R}$

x	0	4	9
$f'(x)$		[+	]
$f(x)$		[$\nearrow$	]

$e^{x}-1>0 \Leftrightarrow e^{x}>1 \Leftrightarrow x>0$

$\Rightarrow f(x) > f(4) = e^{4}-1 > 0 \quad \forall\, x \in [4,9]$ cc tot. $\Rightarrow$

$\Rightarrow$ (2)(A) $\Rightarrow e^{x} > x+1 \quad \forall\, x \in [4,9]$

pt $x \in [2,3] \Rightarrow x^2 \in [4,9]$

$\Rightarrow e^{x^2} > x^2+1 \quad \forall\, x \in [2,3] \Rightarrow$

$\Rightarrow \int_2^3 e^{x^2}\,dx > \int_2^3 x^2+1\,dx$

Met II

Știu că $\boxed{e^{x} > x+1 \quad \forall\, x \in (0,\infty)}$

pt $x \in [2,3] \Rightarrow x^2 \in [4,9] \Rightarrow x^2 > 0 \Rightarrow e^{x^2} > x^2+1 \quad \forall\, x \in [2,3]$

$\Rightarrow \int_2^3 e^{x^2}\,dx > \int_2^3 x^2+1\,dx$

Demonstrați că:

a) $\ln(1+x) < x \quad \forall\, x \in (0,\infty)$

b) $e^{x} > x+1 \quad \forall\, x \in (0,\infty)$

c) $\sin x < x \quad \forall\, x \in (0,\infty)$

d) $x < \operatorname{tg} x \quad \forall\, x \in (0,\infty)$

Rețineți!