Calculați:

- $\int_{-1}^{1} \underline{x^{2017}} + \underset{\sim}{x^2} + \underline{x} + \underset{\sim}{1}\, dx = \int_{-1}^{1} \underbrace{x^{2017} + x}_{\text{impară}}\, dx + \int_{-1}^{1} \underbrace{x^2+1}_{\text{pară}}\, dx =$

$$= 0 + 2\int_0^1 x^2+1\, dx =$$

$$= 2\left(\frac{x^3}{3} + x\right)\Big|_0^1 = 2\left(\frac{1}{3}+1\right) = \frac{8}{3}$$

- $\int_{-\frac{1}{e}}^{\frac{1}{e}} \underbrace{\ln\left|\frac{1+x}{1-x}\right|}_{f(x)} dx = I$

obs $f(-x) = \ln\left|\frac{1-x}{1+x}\right| = \ln\left|\frac{1+x}{1-x}\right|^{-1} = -\ln\left|\frac{1+x}{1-x}\right| = -f(x) \Rightarrow$

$$I = \int_{-\frac{1}{e}}^{\frac{1}{e}} \underbrace{f(x)}_{\text{impară}}\, dx = 0.$$

Calculați:

a) $\int_{-1}^{2} |x|\,dx = \int_{-1}^{1} \underbrace{|x|}_{\text{pară}}\,dx + \int_{1}^{2} |x|\,dx = 2\int_{0}^{1} |x|\,dx + \int_{1}^{2} x\,dx =$

$= 2\int_{0}^{1} x\,dx + \int_{1}^{2} x\,dx = 2\cdot \frac{x^2}{2}\Big|_0^1 + \frac{x^2}{2}\Big|_1^2 =$

$= 2\cdot\frac{1}{2} - 0 + \frac{4}{2} - \frac{1}{2} = \frac{5}{2}$

$$\boxed{\begin{aligned} &\int_{-a}^{a} f(x)\,dx \overset{f\text{ pară}}{=\!=} 2\int_0^a f(x)\,dx \\ &\int_{-a}^{a} f(x)\,dx \underset{\text{impară}}{\overset{f}{=\!=}} 0 \end{aligned}}$$

b) $\int_{-\frac{1}{2}}^{\frac{1}{2}} \underbrace{\ln\left|\frac{1-x}{1+x}\right|}_{f(x)}\,dx = I$

$f(-x) = \ln\left|\frac{1-(-x)}{1+(-x)}\right| = \ln\left|\frac{1+x}{1-x}\right| = \ln\left|\frac{1-x}{1+x}\right|^{-1} = -\ln\left|\frac{1-x}{1+x}\right| = -f(x)$

$\forall\, x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$ int simetric $\Rightarrow f$ impară pe $\left[-\frac{1}{2}, \frac{1}{2}\right] \Rightarrow$

$\Rightarrow I = \int_{-\frac{1}{2}}^{\frac{1}{2}} \underset{\text{impară}}{f(x)}\,dx = 0$

c) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^7 x + \cos x\,dx = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \underbrace{\sin^7 x}_{\text{impară}}\,dx + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \underbrace{\cos x}_{\text{pară}}\,dx =$

$= 0 + 2\int_{0}^{\frac{\pi}{2}} \cos x\,dx = 2\cdot \sin x\Big|_0^{\frac{\pi}{2}} = 2\left(\sin\frac{\pi}{2} - \sin 0\right) = 2$

d) $\int_{-2018}^{2018} \underbrace{x^{2017}}_{\text{impară}}\,dx = 0$

e) $\int_{-2018}^{2018} \underbrace{\frac{x^{2017}}{x^2+1}}_{\text{impară}}\,dx = 0$

f) $\int_0^3 \underbrace{|2x-3|}_{\text{continuă deci integrabilă}} dx = I$

$2x-3 < 0 \Leftrightarrow x < \frac{3}{2} \Rightarrow \quad |2x-3| = \begin{cases} 3-2x, & x < \frac{3}{2} \\ 2x-3, & x \geq \frac{3}{2} \end{cases} \Rightarrow$

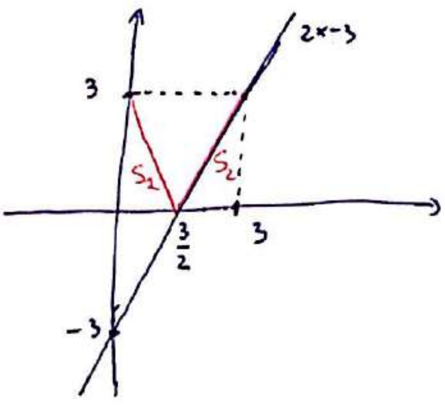
$\Rightarrow I = \int_0^{\frac{3}{2}} |2x-3|\, dx + \int_{\frac{3}{2}}^3 |2x-3|\, dx =$

$\Rightarrow S_1 \qquad \Rightarrow S_2$

$= \int_0^{\frac{3}{2}} 3-2x\, dx + \int_{\frac{3}{2}}^3 2x-3\, dx =$

$= (3x - x^2)\Big|_0^{\frac{3}{2}} + (x^2-3x)\Big|_{\frac{3}{2}}^3 =$

$= \left(\frac{9}{2} - \frac{9}{4}\right) - 0 + (9-9) + \left(\frac{9}{2} - \frac{9}{4}\right) = 2\cdot\frac{9}{4} = \frac{9}{2}$

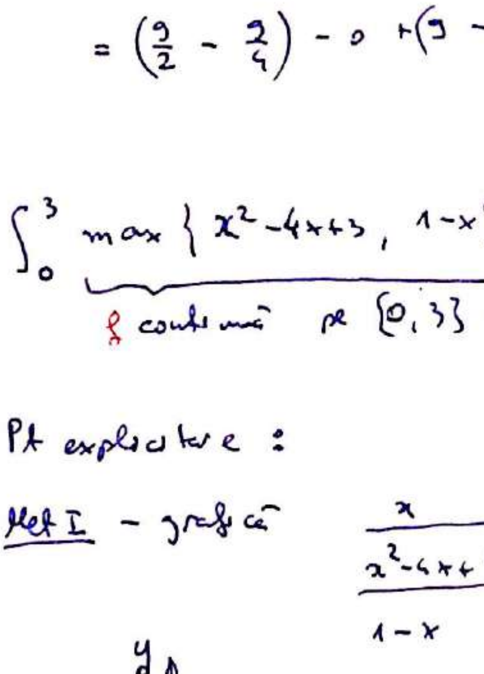


g) $\int_0^3 \underbrace{\max\{x^2-4x+3,\ 1-x\}}_{f \text{ continuă pe } [0,3] \text{ deci integrabilă pe } [0,3]} dx$

Pt explicitare:

Met I - grafică

x	1	2	3
x^2-4x+3	0	-1	0
$1-x$	0	-1	



$\Rightarrow f(x) = \begin{cases} x^2-4x+3, & x \in [0,1) \\ 1-x, & x \in [1,2) \\ x^2-4x+3, & x \in [2,3] \end{cases}$

Met II - algebrică

$x^2-4x+3 < 1-x \Leftrightarrow x^2-3x+2 < 0 \Leftrightarrow x \in (1,2) \Rightarrow f(x) = \begin{cases} x^2-4x+3, & x \in [0,1 \\ 1-x, & x \in (1,2) \\ x^2-4x+3, & x \in [2,3 \end{cases}$

$\Rightarrow I = \int_0^1 x^2-4x+3\, dx + \int_1^2 1-x\, dx + \int_2^3 x^2-4x+3\, dx = \left(\frac{x^3}{3} - 2x^2+3x\right)\Big|_0^1 + \left(x - \frac{x^2}{2}\right)\Big|_1^2 + \left(\frac{x^3}{3} - 2x^2+3\right.$

$= \frac{1}{3} - 2 + 3 + \cancel{2} - 2 - \frac{1}{2} + (9-18+6) - \left(\frac{8}{3} - 8 + \cancel{6}\right) = -\frac{7}{3} - \frac{1}{2} - 16$

h) $\int_0^3 [x]\,dx$

x	0	1	2	3
$[x]$	[0)	[1)	[2)	3

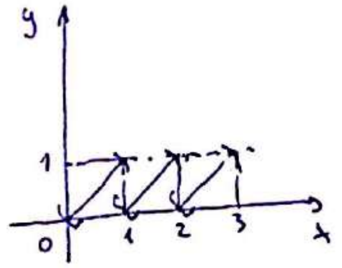
nu e cont dar e cont pe porțiuni și are 3 pcte de disc. de SPEȚA I

1, 2, 3 ⇒ e integrabilă și

$$\int_0^3 [x]\,dx = \int_0^1 0\,dx + \int_1^2 1\,dx + \int_2^3 2\,dx = 0 + x\Big|_1^2 + 2x\Big|_2^3 = 2-1+2(3-2)=3$$

i) $\int_0^3 \{x\}\,dx$

cont pe porțiuni cu nr. finit de pcte de disc de speța I deci integrabilă și



$$\int_0^3 \{x\}\,dx = \int_0^1 \{x\}\,dx + \int_1^2 \{x\}\,dx + \int_2^3 \{x\}\,dx =$$

$$= \int_0^1 x\,dx + \int_1^2 x-1\,dx + \int_2^3 x-2\,dx =$$

$$= \frac{x^2}{2}\Big|_0^1 + \left(\frac{x^2}{2}-x\right)\Big|_1^2 + \left(\frac{x^2}{2}-2x\right)\Big|_2^3 =$$

$$= \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{3}{2}$$

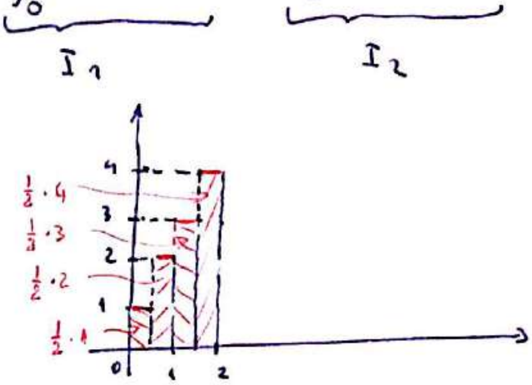
Prefer:

$$\int_0^3 \{x\}\,dx = \int_0^3 x - [x]\,dx = \int_0^3 x\,dx - \int_0^3 [x]\,dx = \frac{x^2}{2}\Big|_0^3 - \dots = \frac{9}{2} - 3 = \frac{3}{2}$$

j) $$I = \int_0^2 \{2x+1\}\,dx = \int_0^2 2x+1 - [2x+1]\,dx = \underbrace{\int_0^2 2x+1\,dx}_{I_1} - \underbrace{\int_0^2 [2x+1]\,dx}_{I_2}$$

$$I_1 = (x^2+x)\Big|_0^2 = 4+2-0 = 6$$

x	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2
$2x+1$	1	2	3	4	5
$[2x+1]$	[1)	[2)	[3)	[4)	



$$I_2 = \int_0^{\frac{1}{2}} 1\,dx + \int_{\frac{1}{2}}^1 2\,dx + \int_1^{\frac{3}{2}} 3\,dx + \int_{\frac{3}{2}}^2 4\,dx = \frac{1}{2}(1+2+3+4) = \frac{1}{2}\cdot\frac{4\cdot 5}{2} = 5$$

$$\Rightarrow I = I_1 - I_2 = 1$$